Computer Algebra Systems: Tools for Developing Algebraic Habits of Mind

EDC's Center for Mathematics Education

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How many irreducible factors are there of $x^n - 1$ over the Integers?

Use your CAS to fill out the table on page 12 of your handout.

How do you do that?

- If you are not in Calculator mode, click the Home button and then "enter" or "1" to select "Add Calculator."
- To factor a polynomial like x³ 1, type factor(x³-1) and hit "enter."





Make Some Conjectures

Ask yourself questions like these:

- Is there a pattern to the number of factors?
- Can you predict what some larger values might be?





CME PROJECT

The next generation of NSF-funded high school mathematics programs



Opinions About the Discipline

...the future well-being of our nation and people depends not just on how well we educate our children generally, but on how well we educate them in mathematics and science specifically.

John Glenn, September, 2000

The only people who need to study calculus are people who want to be calculus teachers.

Bill Cosby, April, 1999





...and about How to Learn It

I'd never use a curriculum that has worked-out examples in the student text.

Nancy McLaughlin CME Project Teacher Advisory Board

I'd never use a curriculum that doesn't have worked-out examples in the student text.

Chuck Garabedian CME Project Teacher Advisory Board



The Utility of Mathematics

Mathematics constitutes one of the most ancient and noble intellectual traditions of humanity. It is an enabling discipline for all of science and technology, providing powerful tools for analytical thought as well as the concepts and language for precise quantitative description of the world around us.

It affords knowledge and reasoning of extraordinary subtlety and beauty, even at the most elementary levels.

RAND Mathematics Study Panel, 2002



What Is the CME Project?

✓ A Brand New, Comprehensive, 4-year Curriculum [™] Problem-Based, Student-Centered Approach **%** "Traditional" **Course Structure**





Algebra 1

Contributors

- EDC's Center for Mathematics Education
- National Advisory Board
- Core Mathematical Consultants
- Teacher Advisory Board
- Field-Test Teachers



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Fundamental Organizing Principle

The widespread utility and effectiveness of mathematics come not just from mastering specific skills, topics, and techniques, but more importantly, from developing the ways of thinking—the *habits of mind*—used to create the results.



"Traditional" course structure: it's familiar but different

- Structured around the sequence of Algebra 1, Geometry, Algebra 2, and Precalculus
- We Uses a variety of instructional approaches
- Focuses on particular mathematical habits
- Uses examples and contexts from many fields
- Organized around mathematical themes
- Student text serves as both a guide and a reference



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CME Project audience:

the (large number of) teachers who...

Want the familiar course structure

Want a problem- and exploration-based program

Want to bring activities to "closure"

Want rigor and accessibility for all



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CME Project Overview Relationship with Texas Instruments



CME Project makes essential use of technology:

- A "function-modeling" language (FML)
- A computer algebra system (CAS)
- An interactive geometry environment



Algebra 1

CME Project Overview Why CAS-Based Technology?

- To make tractable and to enhance many beautiful classical topics, historically considered too technical for high school students.
- To support experiments with algebraic expressions and other mathematical objects in the same way that calculators can be used to experiment with numbers.
- To allow students to build computational models of algebraic objects and structures that have no faithful physical counterparts.



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Comparing Function Definitions

Input: n	Output
0	3
1	8
2	13
3	18
4	23
5	28

Find a closed-form definition *f* and a recursive definition *g* with the table to the left.





Comparing Function Definitions

Input: n	Output
0	6
1	12
2	20
3	30
4	42
5	56

Find a closed-form definition *q* and a recursive definition *r* with the table to the left.





Comparing Function Definitions

Input: n	Output
0	3
1	6
2	12
3	24
4	48
5	96

Find a closed-form definition *d* and a recursive definition *e* with the table to the left.



CME Project

- CME Project Overview
 Problem-Based, Student-Centered Approach
 Comprehensive
 - 4-year Curriculum









Newton's Difference Formula



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CME Project Availability Dates

- Available right now!
 - CME Project Algebra 1 Sampler
- **Available in Spring 2007**
 - CME Project Geometry Sampler
 - CME Project Technology Sampler
- **Available in Fall 2007**
 - Piloting of Algebra 1/Geometry
 - CME Algebra 1 text (November 2007)
 - CME Geometry text (November 2007)
- **Available in Spring 2008**
 - CME Algebra 2 text
- **Available in Summer 2008**
 - CME Precalculus text



PEARSON Prentice Hall

www.phschool.com/cme www.edc.org/cmeproject

