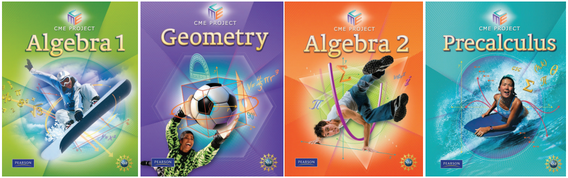

The CME Project

Promoting Mathematical Habits of Mind in High School

Al Cuoco, Wayne Harvey, Sarah Sword, Kevin Waterman

September 27, 2008

1. **Getting Started**
   - What is *The CME Project*?
   - The Habits of Mind Approach
   - Why We Applied to the NSF
   - Design Principles
2. **Habits of Mind**
   - Examples of Mathematical Habits

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Current News

"Many blame predatory lending practices for the recent bank failures. I blame the inability . . . to calculate the effect of a variable interest rate on monthly mortgage payments.

In a culture in which people chuckle at how bad they were in math during their school years, the cost of mathematical illiteracy in this nation is now measured in the tens of billions."

— Neil DeGrasse Tyson
New York Times, September 19, 2008

Opinions about the Discipline

. . . the future well-being of our nation and people depends not just on how well we educate our children generally, but on how well we educate them in mathematics and science specifically.

— John Glenn, September, 2000

The only people who need to study calculus are people who want to be calculus teachers.

— Bill Cosby, April, 1999

. . . AND ABOUT HOW TO LEARN IT

I'd never use a curriculum that has worked-out examples in the student text.

— Nancy, *CME Project* Teacher Advisory Board

I'd never use a curriculum that doesn't have worked-out examples in the student text.

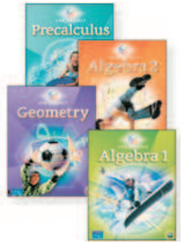
— Chuck, *CME Project* Teacher Advisory Board

Results from Prior Work

- *Connected Geometry* (1993)
- *Mathematical Methods* (1996). Both were
  - Problem-based
  - Student-centered
  - Organized around mathematical thinking
- But. . .
  - Investigations needed closure
  - Students needed a reference
- Enter *The CME Project* (2003)

THE CME PROJECT: BRIEF OVERVIEW

- An NSF-funded coherent 4-year curriculum
- Published by Pearson
- Uses Texas Instruments handheld technology to support mathematical thinking
- Follows the traditional American course structure
- **Organized around mathematical habits of mind**



The Habits of Mind Approach

Mathematics constitutes one of the most ancient and noble intellectual traditions of humanity. It is an enabling discipline for all of science and technology, providing powerful tools for analytical thought as well as the concepts and language for precise quantitative description of the world around us. ***It affords knowledge and reasoning of extraordinary subtlety and beauty, even at the most elementary levels.***

— RAND Mathematics Study Panel, 2002

The Habits of Mind Approach

What mathematicians most wanted and needed from me was ***to learn my ways of thinking****, and not in fact to learn my proof of the geometrization conjecture for Haken manifolds.*

— William Thurston

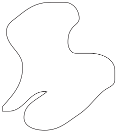
On Proof and Progress in Mathematics

OUR FUNDAMENTAL ORGANIZING PRINCIPLE

The widespread utility and effectiveness of mathematics come not just from mastering specific skills, topics, and techniques, but more importantly, from developing the ways of thinking—the ***habits of mind****—used to create the results.*

—*The CME Project* Implementation Guide, 2008

Habits of Mind: Examples

- Is there a line that cuts the area of  in half?
- Was there a time in your life when your height in inches was equal to your weight in pounds? (*Tom Banchoff*)
- Is the average of two averages the average of the lot?

. . . *more later*. . .

Why We Applied to the NSF

- The field demanded a student-centered program with the traditional American structure.
- That structure allowed us to focus on habits of mind.
- We wanted core involvement of the *entire* mathematical community.
- We had built up decades of experience with classroom-effective methods.
- We wanted a program that helped students bring mathematics into their world.
- We wanted a program with high expectations for students and teachers.

. . . this led to additional core principles.

Additional Core Principles

- **Textured emphasis.** We focus on matters of mathematical substance, being careful to separate them from convention and vocabulary. Even our practice problems are designed so that they have a larger mathematical point.
- **General purpose tools.** The methods and habits that students develop in high school should serve them well in their later work in mathematics and in their post-secondary endeavors.
- **Experience before formality.** Worked-out examples and careful definitions are important, but students need to grapple with ideas and problems *before* they are brought to closure.

Additional Core Principles, continued

- **The role of applications.** What matters is *how* mathematics is applied, not *where* it is applied.
- **A mathematical community.** Our writers, field testers, reviewers, and advisors come from all parts of the mathematics community: teachers, mathematicians, education researchers, technology developers, and administrators.
- **Connect school mathematics to the discipline.** Every chapter, lesson, problem, and example is written with an eye towards how it fits into the landscape of mathematics as a scientific discipline.

Design Principles

Structure of Each Book

- Low threshold, high ceiling
  - Each book has exactly eight chapters
  - Problem sets, investigations, and chapters build from easy access to quite challenging
- Openings and closure
  - *Getting Started*
  - Worked out examples
  - Definitions and theorems are capstones, not foundations
- Coherent and connected
  - Recurring themes, contexts, and methods
  - Small number of central ideas
  - Stress connections between algebra, geometry, analysis, and statistics

These are consistent with recommendations from, for example, NCTM…

Design Principles
Questions from NCTM

- Do curriculum materials for high school mathematics include a central focus on reasoning and sense making that goes beyond the inclusion of isolated supplementary lessons or problems?
- Does the curriculum, whether integrated or following the course sequence customary in the US, develop connections among content areas so that students see mathematics as a coherent whole?
- Is a balance maintained in the areas of mathematics addressed, so that statistics, for example, is more than an isolated unit?
- Does the curriculum emphasize coherence from one course to the next, demonstrating growth in both mathematical content and reasoning?

— NCTM's *Focus on High School Mathematics* (Draft)

Design Principles

Consistent Design Elements

- *Minds in Action*
- *In-Class Experiment*
- *For You to Do*
- *Developing Habits of Mind*
- Projects
- Sidenotes
- Orchestrated problem sets
- Technology support

Design Principles

Example: *Minds in Action* Dialogues

Minds in Action episode 5

Sasha guesses what number Tony chose.

Tony Hey, I'll bet you can't guess my number!

Sasha Guessing again? Okay. Is it 246.3?

Tony No.

Sasha What about 137 and a quarter?

Tony No. I'll give you a big hint, but then you get only one guess. When you square my number, you get 169. Alright, so, what's my number?

Sasha gets out a calculator and fiddles with it for a moment.

Sasha I got it. It's 13.

Tony No! I fooled you.

Design Principles

Example: *Minds in Action* Dialogues

Overheard in a CME Project Classroom

Sara I found your number. It's -5.9.

Zoie Wrong.

Sara It's right! Look.

Sara goes through a process of reversing steps.

See?

Zoie Look.

Zoie goes through steps on her calculator...

Sara Uh-uh! You can't do that! You have to push "equals" every time.

Design Principles

Example: *Minds in Action* Dialogues

Overheard in a CME Project Classroom, cont.

Zoie No! It's the same.

Sara Is it?

Zoie Well, it should be. But it's not.

Sara Why do you think it isn't? Because of "please my dear stuff"... you know... order of operations.

Zoie Oooh. Yeah! So it's not the same.

Sara Eew. We sound like Tony and Sasha.

General Mathematical Habits

- Performing thought experiments
- Finding and explaining patterns
- Creating and using representations
- Generalizing from examples
- Expecting mathematics to make sense

Algebraic Habits of Mind

- Seeking regularity in repeated calculations
- "Chunking" (changing variables in order to hide complexity)
- Reasoning about and picturing calculations and operations
- Extending operations to preserve rules for calculating
- Purposefully transforming and interpreting expressions
- Seeking and specifying structural similarities

An Example from Algebra

Factoring monic quadratics:

"Sum-Product" problems

$$x^2 + 14x + 48$$

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

so...

Find two numbers whose sum is 14 and whose product is 48.

$$(x + 6)(x + 8)$$

An Example from Algebra

What about this one?

$$49x^2 + 35x + 6$$

$$\begin{aligned} 49x^2 + 35x + 6 &= (7x)^2 + 5(7x) + 6 \\ &= \clubsuit^2 + 5\clubsuit + 6 \\ &= (\clubsuit + 3)(\clubsuit + 2) \\ &= (7x + 3)(7x + 2) \end{aligned}$$

An Example from Algebra

What about this one?

$$6x^2 + 31x + 35$$

$$\begin{aligned} 6(6x^2 + 31x + 35) &= (6x)^2 + 31(6x) + 210 \\ &= \clubsuit^2 + 31\clubsuit + 210 \\ &= (\clubsuit + 21)(\clubsuit + 10) \\ &= (6x + 21)(6x + 10) \\ &= 3(2x + 7) \cdot 2(3x + 5) \\ &= 6(2x + 7)(3x + 5) \quad \textit{so} \ldots \end{aligned}$$

$$6(6x^2 + 31x + 35) = 6(2x + 7)(3x + 5)$$

An Example from Algebra

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&= (6x + 21)(6x + 10) \\
&= 3(2x + 7) \cdot 2(3x + 5) \\
&= 6(2x + 7)(3x + 5) \quad \textit{so} \ldots
\end{aligned}$$

$$\cancel{6}(6x^2 + 31x + 35) = \cancel{6}(2x + 7)(3x + 5)$$

An Example from Algebra

What about this one?

$$6x^2 + 31x + 35$$

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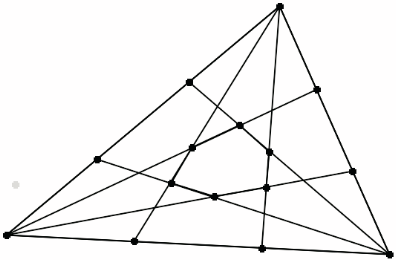
$$6x^2 + 31x + 35 \;=\; (2x + 7)(3x + 5)$$

Analytic/Geometric Habits of Mind

- Reasoning by continuity
- Seeking geometric invariants
- Looking at extreme cases
- Passing to the limit
- Using approximation

An Example from Geometry



What kids can do. . .

FINAL EDITION

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THE SUN

LATEST NEWS & SPORTS

TUESDAY, DECEMBER 20, 1994 — HOME DELIVERY: 25¢ — NEWSSTAND: 50¢ — BALTIMORE, MARYLAND

SOMETHING NEW IN A TRIANGLE

Patapsco High student's hunch points to theorem

By Mary Maushard
Sun Staff Writer

Ryan Morgan would have gotten an "A" in geometry even if he hadn't unearthed a mathematical treasure.

But the persistent Patapsco High School sophomore pushed a hunch into a theory. He calls it Morgan's Conjecture, and is hoping it will soon be Morgan's Theorem.

In geometric circles, developing a theorem is a big deal — especially if you're only 15.

Ryan's teacher at Patapsco High, Frank Nowosielski, has been teaching 20 years and has never had a student discover a theorem — a mathematical statement that can be proved universally true.

Towson State University math professor Robert B. Hanson never had a high school student present a possible theorem to his faculty seminar — until Ryan did it last spring.

"Ryan's really done something pretty fantastic," said Mr. Nowosielski, who taught Ryan's ninth-grade geometry class for gifted and talented students las' year and now teaches at the Carver Center for Arts and Technology in Towson.

"How many kids in the world have done this? He saw something and he didn't quit. He's a special kid," Mr. Nowosielski said.

What did Ryan see?

Initially, he sa v a triangle, each side divided into thirds. Lines drawn from those segments to the vertices [the corners] formed a hexagon inside the triangle. The area of the hexagon is one-tenth the area of the triangle. This is known as Marion's

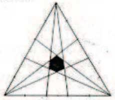
See THEOREM, 18A

JED KIRSCHBAUM/SUN STAFF PHOTO

Ryan Morgan worked many days after school in the computer lab to develop his conjecture, which is displayed on the screen.

MORGAN'S CONJECTURE

Developed by Ryan Morgan, 15, of Patapsco High School:



When the sides of a triangle are n-sected, and n represents any odd integer greater than 1, and segments are drawn from the vertices to these new points, there will be a hexagon present in the interior of the triangle (shaded area). There will always be a constant ratio between the area of the hexagon to the area of the original triangle.

SUN STAFF GRAPHIC

Modeling with Functions

Example 1

Topic: Fitting functions to tables

Habits of Mind:

- Generalizing from examples
- Reasoning about calculations
- Abstracting regularity from repeated calculations

Modeling with Functions

Example 1: Fitting Functions to Tables

In Algebra 1

Find functions that agree with each table:

Input: n	Output
0	3
1	8
2	13
3	18
4	23

Input: n	Output
0	1
1	2
2	5
3	10
4	17

Modeling with Functions

Example 1: Fitting Functions to Tables

In Algebra 1 and Algebra 2

Build a calculator model of a function that agrees with the table:

Input: n	Output
0	3
1	8
2	13
3	18
4	23

- $f(n) = 5n + 3$
- $g(n) = \begin{cases} 3 & \text{if } n = 0 \\ g(n-1) + 5 & \text{if } n > 0 \end{cases}$

Question:

$$f \stackrel{?}{=} g$$

Modeling with Functions

Example 1: Fitting Functions to Tables

In Precalculus

$$f(n) = 5n + 3 \qquad \text{and} \qquad g(n) = \begin{cases} 3 & \text{if } n = 0 \\ g(n-1) + 5 & \text{if } n > 0 \end{cases}$$

OK. $f(254) = g(254)$. Is $f(255)$=$g(255)$?

$$\begin{aligned} g(255) &= g(254) + 5 && \text{(this is how } g \text{ is defined)} \\ &= f(254) + 5 && \text{(CSS)} \\ &= (5 \cdot 254 + 3) + 5 && \text{(this is how } f \text{ is defined)} \\ &= (5 \cdot 254 + 5) + 3 && \text{(BR)} \\ &= 5(254 + 1) + 3 && \text{(BR)} \\ &= 5(255) + 3 && \text{(arithmetic)} \\ &= f(255) && \text{(this is how } f \text{ is defined)} \end{aligned}$$

Modeling with Functions

Example 1: Fitting Functions to Tables

In Precalculus

$$f(n) = 5n + 3 \qquad \text{and} \qquad g(n) = \begin{cases} 3 & \text{if } n = 0 \\ g(n-1) + 5 & \text{if } n > 0 \end{cases}$$

OK. Suppose $f(321) = g(321)$. Is $f(322)=g(322)$?

$$\begin{aligned} g(322) &= g(321) + 5 && \text{(this is how } g \text{ is defined)} \\ &= f(321) + 5 && \text{(CSS)} \\ &= (5 \cdot 321 + 3) + 5 && \text{(this is how } f \text{ is defined)} \\ &= (5 \cdot 321 + 5) + 3 && \text{(BR)} \\ &= 5(321 + 1) + 3 && \text{(BR)} \\ &= 5(322) + 3 && \text{(arithmetic)} \\ &= f(322) && \text{(this is how } f \text{ is defined)} \end{aligned}$$

Modeling with Functions

Example 1: Fitting Functions to Tables

In Precalculus

$$f(n) = 5n + 3 \qquad \text{and} \qquad g(n) = \begin{cases} 3 & \text{if } n = 0 \\ g(n-1) + 5 & \text{if } n > 0 \end{cases}$$

OK. Suppose $f(n-1) = g(n-1)$. Is $f(n)=g(n)$?

$$\begin{aligned} g(n) &= g(n-1) + 5 && \text{(this is how } g \text{ is defined)} \\ &= f(n-1) + 5 && \text{(CSS)} \\ &= (5(n-1) + 3) + 5 && \text{(this is how } f \text{ is defined)} \\ &= (5(n-1) + 5) + 3 && \text{(BR)} \\ &= 5(n-1+1) + 3 && \text{(BR)} \\ &= 5n + 3 && \text{(arithmetic)} \\ &= f(n) && \text{(this is how } f \text{ is defined)} \end{aligned}$$

Modeling with Functions

Example 2

Topic: Monthly payments on a loan

Habits of Mind:

- Expecting mathematics to make sense
- Reasoning about calculations
- Abstracting regularity from repeated calculations
- Chunking

Modeling with Functions

Example 2: Monthly Payments on a Loan

Suppose you want to buy a car that costs $10,000. You don't have much money, but you can put $1000 down and pay $350 per month. The interest rate is 5%, and the dealer wants the loan paid off in three years. What kind of car can you buy?

This leads to the question

"How does a bank figure out the monthly payment on a loan?"

or

"How does a bank figure out the balance you owe at the end of the month?"

Modeling with Functions

Example 2: Monthly Payments on a Loan

Take 1

What you owe at the end of the month is what you owed at the start of the month minus your monthly payment.

$$b(n, m) = \begin{cases} 9000 & \text{if } n = 0 \\ b(n-1, m) - m & \text{if } n > 0 \end{cases}$$

Modeling with Functions

Example 2: Monthly Payments on a Loan

Take 2

What you owe at the end of the month is what you owed at the start of the month, ***plus $\frac{1}{12}$ of the yearly interest on that amount****, minus your monthly payment.*

$$b(n, m) = \begin{cases} 9000 & \text{if } n = 0 \\ b(n-1, m) + \frac{.05}{12} b(n-1, m) - m & \text{if } n > 0 \end{cases}$$

Students can then use successive approximation to find m so that

$$b(36, m) = 0$$

Modeling with Functions

Example 2: Monthly Payments on a Loan

Except . . .

$$b(4) = b(3) + \frac{.05}{12} \cdot b(3) - 250$$

$$b(2) + \frac{.05}{12} \cdot b(2) - 250 \qquad\qquad b(2) + \frac{.05}{12} \cdot b(2) - 250$$

It takes too much !$#& work.

Modeling with Functions

Example 2: Monthly Payments on a Loan

Take 3: Algebra to the rescue!

$$b(n, m) = \begin{cases} 9000 & \text{if } n = 0 \\ b(n-1, m) + \frac{.05}{12} b(n-1, m) - m & \text{if } n > 0 \end{cases}$$

becomes

$$b(n, m) = \begin{cases} 9000 & \text{if } n = 0 \\ \left(1 + \frac{.05}{12}\right) b(n-1, m) - m & \text{if } n > 0 \end{cases}$$

Students can *now* use successive approximation to find m so that

$$b(36, m) = 0$$

Modeling with Functions

Example 2: Monthly Payments on a Loan

Project: Pick an interest rate and keep it constant. Suppose you want to pay off a car in 24 months. Investigate how the monthly payment changes with the cost of the car:

Cost of car (in thousands of dollars)	Monthly payment
10	
11	
12	
13	
14	
15	
⋮	⋮

Modeling with Functions

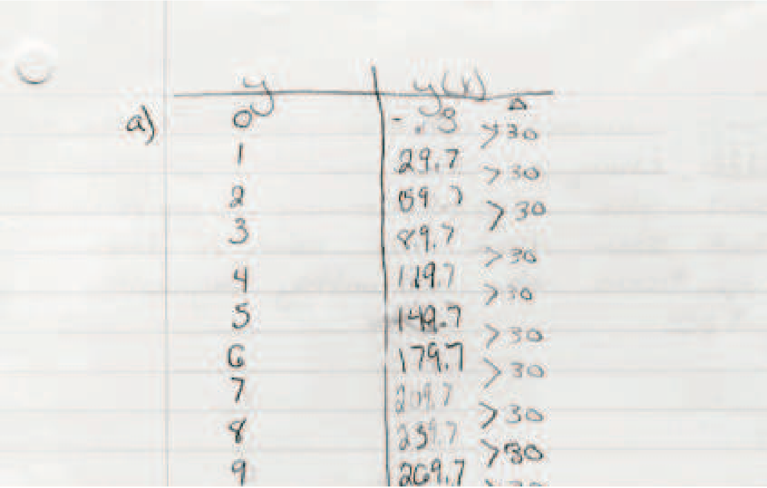
Example 2: Monthly Payments on a Loan

Cost of car (in thousands of dollars)	Monthly payment
10	
11	
12	
13	
14	
15	
⋮	⋮

Describe a pattern in the table. Use this pattern to find either a closed form or a recursive rule that lets you calculate the monthly payment in terms of the cost of the car in thousands of dollars. Model your function with your CAS and use the model to find the monthly payment on a $26000 car.

Modeling with Functions

Example 2: Monthly Payments on a Loan



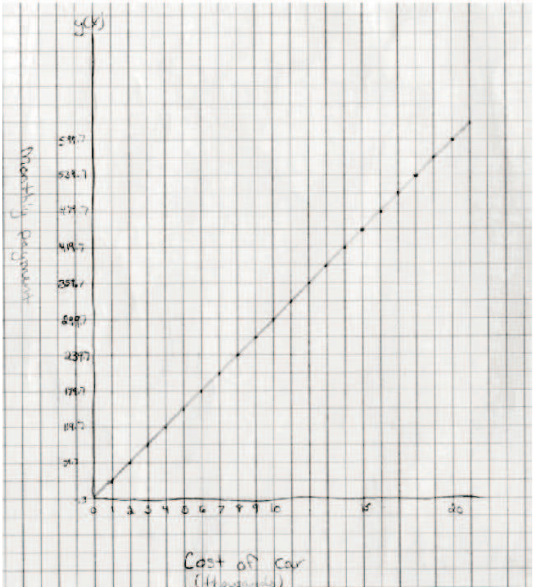
Modeling with Functions

Example 2: Monthly Payments on a Loan

- I changed the amount of the cost of the car then I changed the monthly payment until I found the right monthly payment.
- I found that each time the cost of the car went up $1000 the monthly payment went up $30.

Modeling with Functions

Example 2: Monthly Payments on a Loan



Modeling with Functions

Example 2: Monthly Payments on a Loan

Students can use a CAS to model the problem *generically*: the balance at the end of 36 months with a monthly payment of m can be found by entering

$$b(36, m)$$

in the calculator:

But *why* is it linear?

Modeling with Functions

Example 2: Monthly Payments on a Loan

But *why* is it linear?

Suppose you borrow \$12000 at 5% interest. Then you are experimenting with this function:

$$b(n, m) = \begin{cases} 12000 & \text{if } n = 0 \\ (1 + \frac{.05}{12}) \cdot b(n-1, m) - m & \text{if } n > 0 \end{cases}$$

Notice that

$$1 + \frac{.05}{12} = \frac{12.05}{12}$$

Call this number q. So, the function now looks like:

$$b(n, m) = \begin{cases} 12000 & \text{if } n = 0 \\ q \cdot b(n-1, m) - m & \text{if } n > 0 \end{cases}$$

where q is a constant (chunking, again).

Modeling with Functions

Example 2: Monthly Payments on a Loan

Then at the end of n months, you could unstack the calculation as follows:

$$
\begin{aligned}
b(n, m) &= q \cdot b(n-1, m) - m \\
&= q\left[q \cdot b(n-2, m) - m\right] - m \\
&\quad = q^2 \cdot b(n-2, m) - qm - m \\
&= q^2\left[q \cdot b(n-3, m) - m\right] - qm - m \\
&\quad = q^3 \cdot b(n-3, m) - q^2 m - qm - m \\
&\quad \vdots \\
&= q^n \cdot b(0, m) - q^{n-1} m - q^{n-2} m - \cdots - q^2 m - qm - m \\
&= 12000 \cdot q^n - m(q^{n-1} + q^{n-2} + \cdots + q^2 + q + 1)
\end{aligned}
$$

Modeling with Functions

Example 2: Monthly Payments on a Loan

Precalculus students know (very well) the "cyclotomic identity:"

$$q^{n-1} + q^{n-2} + \cdots + q^2 + q + 1 = \frac{q^n - 1}{q - 1}$$

Applying it, you get

$$\begin{aligned} b(n, m) &= 12000 \cdot q^n - m(q^{n-1} + q^{n-2} + \cdots + q^2 + q + 1) \\ &= 12000\, q^n - m\frac{q^n - 1}{q - 1} \end{aligned}$$

Setting $b(n, m)$ equal to 0 gives an explicit relationship between m and the cost of the car...

Modeling with Functions

Example 2: Monthly Payments on a Loan

$$m = 12000\,\frac{(q-1)q^n}{q^n-1}$$

or, in general,

$$\text{monthly payment} = \text{cost of car} \times \frac{(q-1)q^n}{q^n-1}$$

where n is the term of the loan and

$$q = 1 + \frac{\text{interest rate}}{12}$$

Building Equations
Example 1

Topic: Word problems

Habits of Mind:

- Abstracting regularity from repeated calculations

Building Equations

Example 1: Word Problems

The dreaded algebra word problem

Mary drives from Boston to Chicago, and she travels at an average rate of 60 MPH on the way down and 50 MPH on the way back. If the total trip takes 36 hours, how far is Boston from Chicago?

Why is this so difficult for students?

- Reading level
- Context

Building Equations

Example 1: Word Problems

But there must be more to it. Compare. . .

Mary drives from Boston to Chicago, and she travels at an average rate of 60 MPH on the way down and 50 MPH on the way back. *If the total trip takes 36 hours, how far is Boston from Chicago?*

with

Mary drives from Boston to Chicago, and she travels at an average rate of 60 MPH on the way down and 50 MPH on the way back. *If Boston is 1000 miles from Chicago, how long did the trip take?*

"The difficulty lies in setting up the equation, not solving it."

Building Equations
Example 1: Word Problems

This led to the Guess-Check-Generalize method:

- Take a guess, say 1200 miles.
- Check it:
  - $\frac{1200}{60} = 20$
  - $\frac{1200}{50} = 24$
  - $20 + 24 \neq 36$
- That wasn't right, but that's okay—
  just keep track of your steps.
- Take another guess, say 1000, and check it:

$$\frac{1000}{60} + \frac{1000}{50} \stackrel{?}{=} 36$$

Building Equations

Example 1: Word Problems

- Keep it up, until you get a "guess checker"

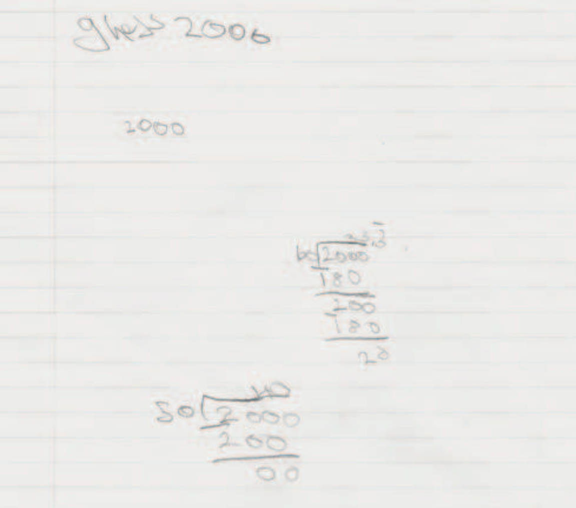
$$\frac{\text{guess}}{60} + \frac{\text{guess}}{50} \stackrel{?}{=} 36$$

- The equation is

$$\frac{x}{60} + \frac{x}{50} = 36$$

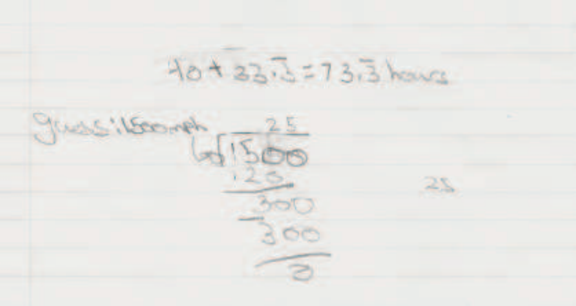
Building Equations

Example 1: Word Problems



Building Equations

Example 1: Word Problems



Building Equations

Example 1: Word Problems

$$50\overline{)1500}\quad 30$$

$$-150$$

$$00$$

$25 + 30 = 55$ hrs

$((\text{guess}) \div 60) + (\text{guess} \div 50) = 36$

$(x \div 60) + (x \div 50) = 36$

Building Equations
Example 2

Topic: Equations of lines

Habits of Mind:

- Abstracting regularity from repeated calculations

Building Equations

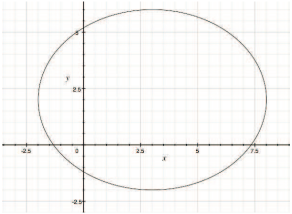
Example 2: Equations of Lines

Graph

$$16\,x^2 - 96\,x + 25\,y^2 - 100\,y - 156 = 0$$

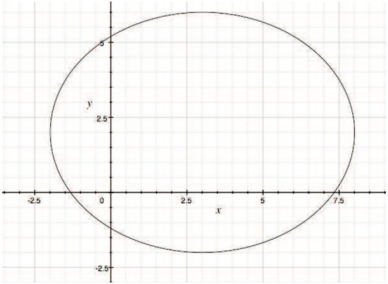
$$16\,x^2 - 96\,x + 25\,y^2 - 100\,y - 156 = 0 \Rightarrow \frac{(x-3)^2}{25} + \frac{(y-2)^2}{16} = 1$$

$$\frac{(x-3)^2}{25} + \frac{(y-2)^2}{16} = 1 \Rightarrow$$



Building Equations

Example 2: Equations of Lines



Is $(7.5, 3.75)$ on the graph?

This led to the idea that "equations are point testers."

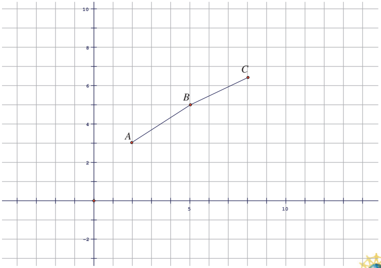
Building Equations

Example 2: Equations of Lines

Why is “linearity” so hard for students?

- Slope is defined initially *between two points*: $m(A, B)$

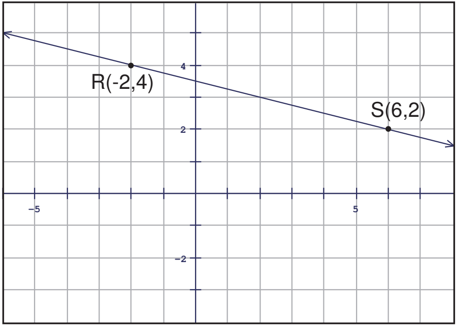
Basic assumption: A, B, and C are collinear $\Leftrightarrow m(A, B) = m(B, C)$



Building Equations

Example 2: Equations of Lines

What is the equation of the line ℓ that goes through $R(-2, 4)$ and $S(6, 2)$?



Try some points, keeping track of the steps…

Building Equations

Example 2: Equations of Lines

Minds in Action episode 14

Sasha and Tony are trying to find the equation of the line ℓ that goes through points $R(-2, 4)$ and $S(6, 2)$.

Sasha To use a point-tester, we first need to find the slope between R and S.

Tony goes to the board and writes

$$m(R, S) = \frac{2-4}{6-(-2)} = \frac{-2}{8} = -\frac{1}{4}.$$

Tony It's $-\frac{1}{4}$.

Sasha Okay. Now, we want to test some point, say P. We want to see whether the slope between that point and one of the first two, say R, is equal to $-\frac{1}{4}$. If it is, that point is on ℓ. So our test is $m(P, R) \stackrel{?}{=} -\frac{1}{4}$.

It doesn't matter which point you choose as the base point. Either point R or point S will work.

Building Equations

Example 2: Equations of Lines

- Test $P = (1, 1)$:
  $m(P, R) = \frac{1-4}{1-(-2)} \stackrel{?}{=} -\frac{1}{4} \Rightarrow$ Nope
- Test $P = (3, 2)$:
  $m(P, R) = \frac{3-4}{2-(-2)} \stackrel{?}{=} -\frac{1}{4} \Rightarrow$ Yup
- Test $P = (7, 2)$:
  Let's see how Tony and Sasha finish this problem.

Building Equations
Example 2: Equations of Lines

Tony Let's guess and check a point first, like $P(7, 2)$. Tell me everything you do so I can keep track of the steps.

Sasha Well, the slope between $P(7, 2)$ and $R(-2, 4)$ is $m(P, R) = \frac{2-4}{7-(-2)} = \frac{-2}{9} = -\frac{2}{9}$. This slope is different, so P isn't on ℓ. Maybe we should use a variable point.

Tony How do we do that?

Sasha A point has two coordinates, right? So use two variables. Say P is (x, y).

Tony Then the slope from P to R is $m(P, R) = \frac{y-4}{x-(-2)} = \frac{y-4}{x+2}$. The test is $\frac{y-4}{x+2} = -\frac{1}{4}$.

So, that must be the equation of the line ℓ.

Notice how Sasha switches to letters. She uses x for point P's x-coordinate. She uses y for point P's y-coordinate.

GEOMETRY AND ANALYSIS

EXAMPLE

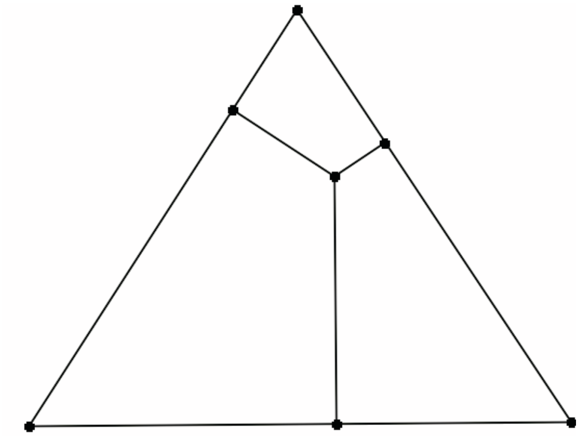
Topic: Invariants in triangles

Habits of Mind:

- Reasoning by continuity
- Reasoning about calculations

Geometry and Analysis

Example: Invariants in Triangles



Arithmetic to Algebra

Example

Topic: Factoring

Habits of Mind:

- Seeking structural similarities

Arithmetic to Algebra
Example: Factoring

The *CMP* Factor Game

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30

Arithmetic to Algebra

Example: Factoring

The *CME Project* Factor Game

$x-1$	x^2-1	x^3-1	x^4-1	x^5-1
x^6-1	x^7-1	x^8-1	x^9-1	$x^{10}-1$
$x^{11}-1$	$x^{12}-1$	$x^{13}-1$	$x^{14}-1$	$x^{15}-1$
$x^{16}-1$	$x^{17}-1$	$x^{18}-1$	$x^{19}-1$	$x^{20}-1$
$x^{21}-1$	$x^{22}-1$	$x^{23}-1$	$x^{24}-1$	$x^{25}-1$
$x^{26}-1$	$x^{27}-1$	$x^{28}-1$	$x^{29}-1$	$x^{30}-1$

CONCLUSION

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