THE *COMMON CORE STATE STANDARDS FOR MATHEMATICAL PRACTICE* AND THE *CME PROJECT*

In a real sense, the entire *CME Project* is *about* mathematical practice. Our construct of *mathematical habits of mind*, the core organizing principle of the program, is aimed at helping students develop precisely the kind of mathematical practices described in the *CCSS*.

Make sense of problems and persevere in solving them. The "experience before formality" principle in the *CME Project* has sense making as one of its main goals. We see to it that definitions and theorems are capstones, not foundations, and that students spend plenty of time playing with ideas in simple and transparent cases before the ideas are formalized and made precise. One device for helping them do this is the recurring *Minds in Action* feature—dialogues in which students model mathematical discussions, giving examples of how students gradually refine insights through perseverance and through attempts to communicate with others. Another example: in *CME Project*, algebraic expressions are purposefully transformed so that students can see where an expression is zero or where it is optimized. Stamina is developed in several ways—recurring themes throughout the program and open-ended, long-term projects at the end of every chapter.

Reason abstractly and quantitatively. The whole "contextualize-decontextualize" dialectic is another major theme of the *CME Project*. In algebra 1, variables start out as placeholders for numbers, and problems always refer back to the quantities that are represented. As the course (and the program) progresses, the symbols become contexts in their own right, so that, for example, the system of polynomials is eventually studied as a structure with its own internal logic. This approach is echoed in the recently released NCTM monograph *Reasoning and Sense Making in High school Mathematics*:

> *Although a long-term goal of algebraic learning is a fluid, nearly automatic facility with manipulating algebraic expressions that might seem to resemble what is often called "mindless manipulation," this ease can be best achieved by first learning to pay close attention to interpreting expressions, both at a formal level and as statements about real-world situations.*

Construct viable arguments and critique the reasoning of others. The roles of proof and justification—as means for establishing results, communicating ideas, and discovering new conjectures—is emphasized in all four courses. Students not only learn to read and understand logical arguments, they learn to construct such arguments for themselves. They learn to develop proofs in algebra by looking at the structure of numerical calculations, and they study explicit and general purpose methods for constructing geometric proofs, combinatorial proofs, and proofs by mathematical induction. A recurring type of problem across the program ("'What's Wrong Here") asks students to critique and repair errors in reasoning.

Model with mathematics. Students in the *CME Project* build mathematical models using functions, equations, graphs, tables, and technology. They learn how to abstract from data to build equations that model a host of phenomena. They build geometric diagrams to represent probability calculations. They use functions and spreadsheets to calculate payments on a loan. They also model mathematical phenomena, building computational models of mathematical functions (including recursively defined functions), and then applying their models to contexts ranging from monthly

payments to Pythagorean triples. They also use mathematics to build approximate models for data; one of the distinguishing features of the *CME Project* is that students learn how to calculate a line of best fit using quadratic functions.

Use appropriate tools strategically. Students in the *CME Project* use an array of tools ranging from physical devices to experiment with geometric optimization problems to technological tools. Technology is integrated throughout the program, with an appendix at the end of each book keyed to the TI-Nspire handhelds. Specifically, student in the program use a function modeling language and a spreadsheet to build computational models of functions, a dynamic geometry environment to develop reasoning by continuity, and a computer algebra system to support algebraic habits of mind. The technology is used in three ways

- to provide students a platform for *experimenting* with mathematical objects.
- To make tractable and to enhance many classical topics, historically considered too technical for high school students, by *reducing computational overhead*.
- To allow students to build computational models of mathematical objects that have no faithful physical counterparts, *highlighting similarities in structure*.

Attend to precision. Students not only learn to use precise definitions, they learn to *formulate* them as a means to communicate with others. The *Minds in Action* dialogues depict students as they wrestle with how to say what they know in precise mathematical language. Another design feature is that students develop vocabulary as its needed, so that, for example, when formal polynomials are introduced in Algebra 1, they are introduced to the terminology of monomial, degree, and coefficient as a language to state things in a precise way, and they learn to distinguish convention in this language (the coefficients of $ax + b$ are usually taken to be a and b) from well-defined notions like the meaning of "degree." They learn to critique a proposed definition by seeking counterexamples and by trying cases to see if the definition captures what they want it to capture.

Look for and make use of structure. Students look for regularity and structure in almost every lesson of the *CME Project*. Right from day 1 in Algebra 1, they start looking for hidden structure in the tables of arithmetic. Our whole approach to completing the square, "chunking" to remove terms in the factoring lessons, and transforming area formulas is devoted to seeing and making use of structure in algebraic expressions. Students also build mathematical structures in the more traditional sense: in Algebra 2, they study the algebra of functions, and they are introduced to complex numbers as polynomials with an extra simplification rule. In Precalculus, they create groups (without calling them that) of roots of unity via "remainder arithmetic" with polynomials.

Look for and express regularity in repeated reasoning. This is an explicit habit of mind that's given serious attention in the *CME Project*, especially in the algebra courses. Our approach to finding equations and functions that model situations, to finding equations of lines, to fitting functions to data, and to finding and establishing algebraic identities all revolve around this practice. Often, students look at a problem and have no idea how to start. A common mantra in the *CME Project* is "try it with numbers." Mathematics is open to experiment, and general results, or at least conjectures, often spring from trying examples, looking for regularity, and seeking what seem to be general trends. For example, we ask algebra 1 students to think about which positive integers can be represented as a difference of two perfect squares. This can lead to a lovely result, but there's another reason to include it in an algebra course: it helps students develop the habit of generalizing from examples. This practice is a cornerstone of our program, a general purpose habit that will serve students well in all their technical endeavors.