

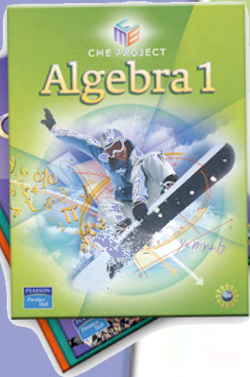
Curriculum and Technology: Evolving Together

Kevin Waterman



Education Development Center

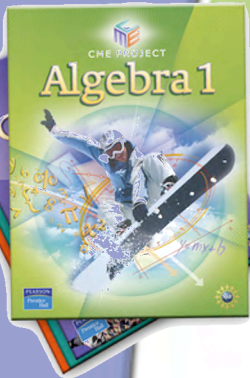
Gosia Brothers

Texas Instruments



Curriculum and Technology: Evolving Together

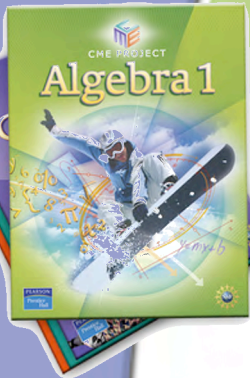
-  New technology features influence curriculum
-  New curriculum ideas drive technology innovation



Defining Recursive Functions

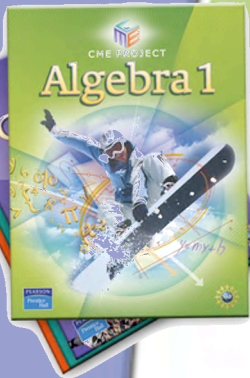
$$f(n) = \begin{cases} 3, & n = 0 \\ f(n - 1) + 5, & n > 0 \end{cases}$$

-  On the TI-89
-  On the TI-Nspire™ Handhelds



Recursive Functions on the TI-89

```
f(n)
Func
If n=0 Then
    Return 3
Else
    Return f(n-1) + 5
EndIf
EndFunc
```



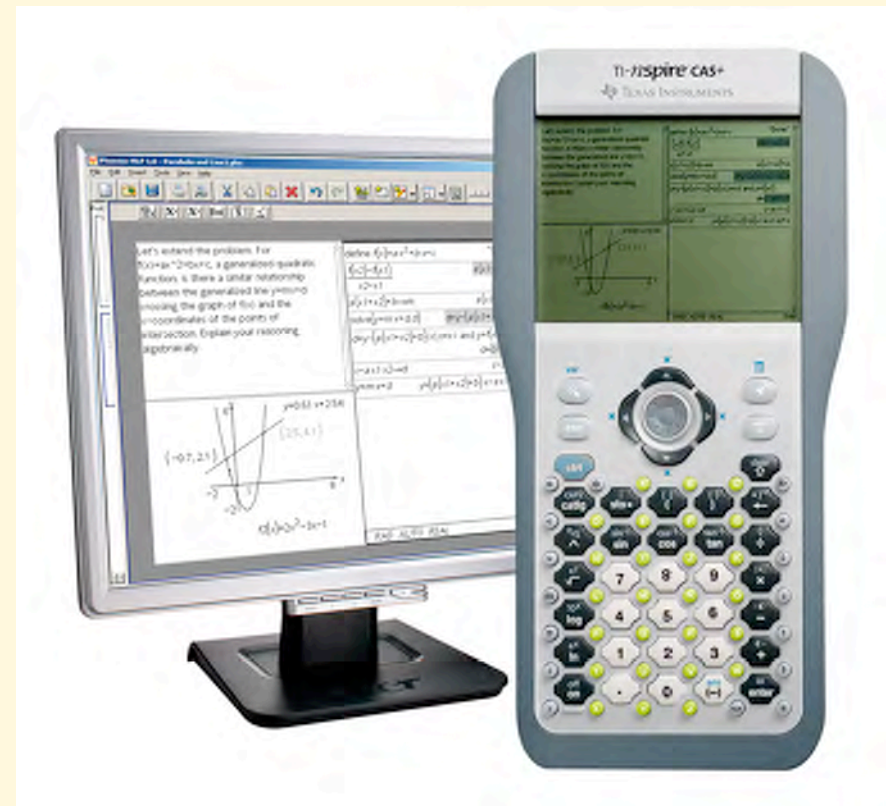
Recursive Functions on the TI-Nspire

- Entered in a template that looks just like what you write or see in a book
- Ease of creation prompted us to introduce recursive definitions in Algebra 1

Integrated Learning Technology

TI-*n*spire delivers the solution educators have been asking for:

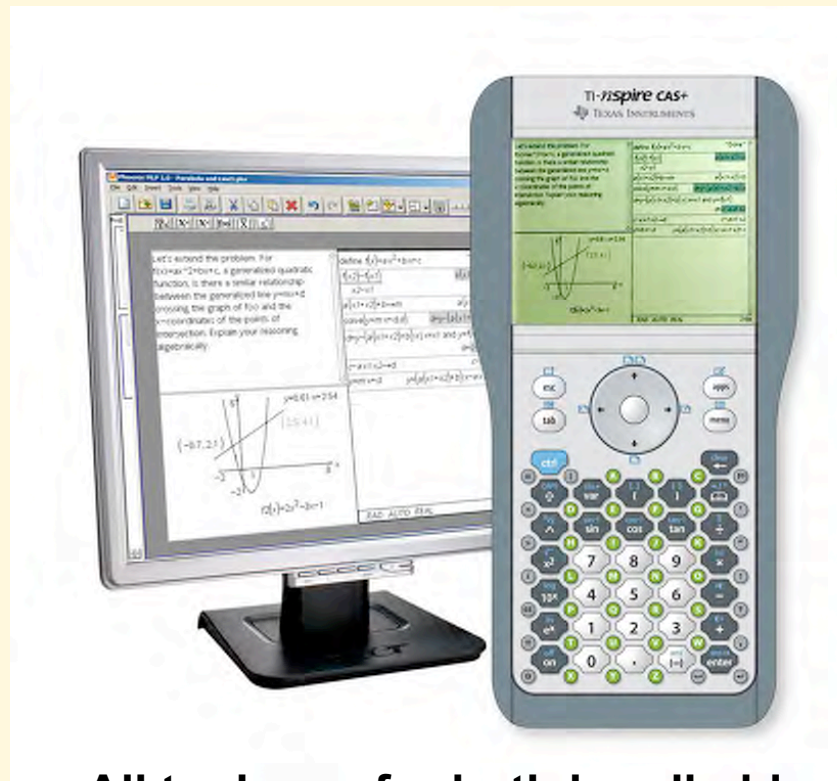
- Multiple representations, dynamically linked, encouraging multiple approaches to solving a problem and expressing solutions.
- Active documents can be saved, recalled, revised, and transferred electronically
- An optimal tool for concept and skill development in secondary Mathematics.
- Integrated with classroom technology using familiar user interface elements
- Grounded in Mathematics research



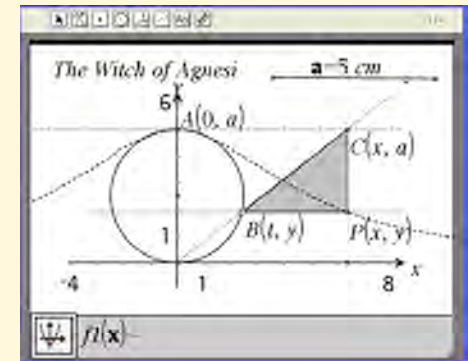
TI-Nspire™ Family Technology – Bringing It All Together

Calculator

Notes



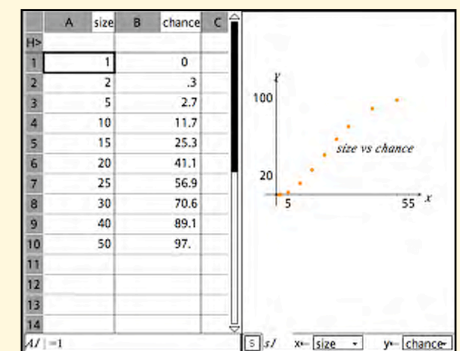
All tools are for both handheld and computer: same functionality, same experience.



Graphs and Geometry

Data Collection

Lists and Spreadsheet



TI-*n*spire™ Products



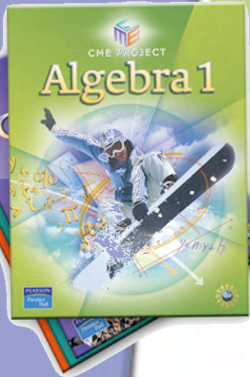
TI-*n*spire™ CAS



TI-*n*spire™



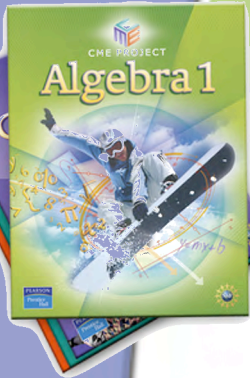
TI-*n*spire™ Plus w/
TI-84 Keypad



Geometry Experimentation

Estimate slope of a curve at a point:

- 📊 On Graphing Calculators
 - 📊 Graph equation
 - 📊 Zoom in until graph “looks linear”
- 📊 On TI-Nspire Handheld
 - 📊 Graph equation
 - 📊 Construct two points and draw line
 - 📊 Move one point toward the other

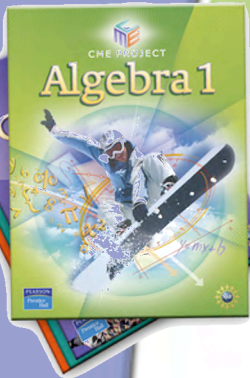


Geometry Experimentation

Estimate the slope of

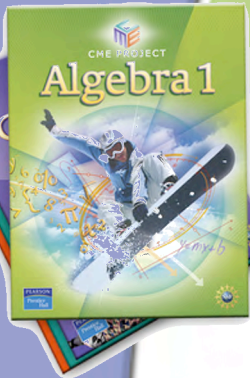
$$y = x^3 - x + 1$$

at point (1,1).



Linking Geometry to Algebra



You can determine the equation of the secant of $y = x^3 - x + 1$ through points $(1, 1)$ and $(2, 7)$ by dividing $x^3 - x + 1$ by $(x - 1)(x - 2)$. The remainder defines the line.



Linking Geometry to Algebra

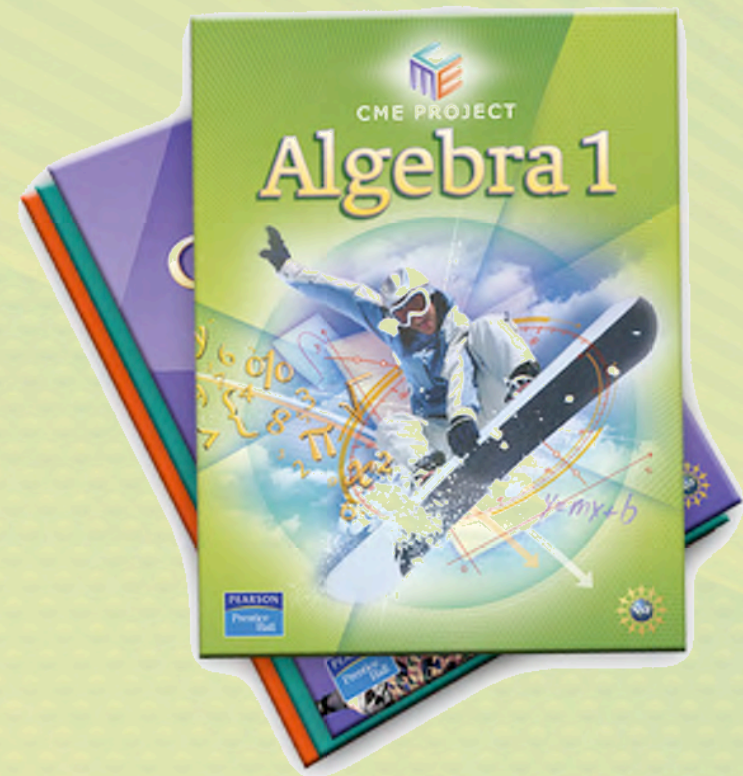
You can determine the equation of the line tangent to $y=x^3 - x + 1$ at point $(1,1)$ by dividing by $(x - 1)^2$. The remainder defines the line.

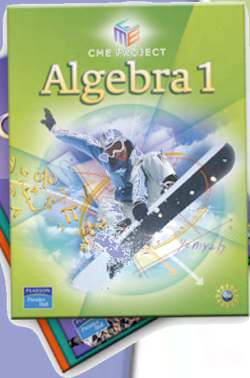
There are two ways to find the remainder:

-  Divide the polynomials and use propFrac
-  Rewrite the original polynomial from base x to base $(x - 1)$

What is the CME Project?






- ❏ A Brand New, Comprehensive, 4-year Curriculum
- ❏ NSF-funded
- ❏ Problem-Based, Student-Centered Approach
- ❏ “Traditional” Course Structure

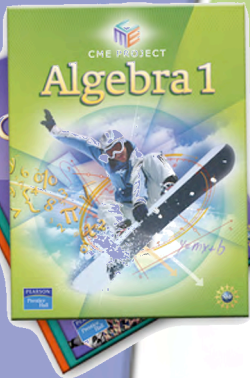




CME Project Overview

Contributors

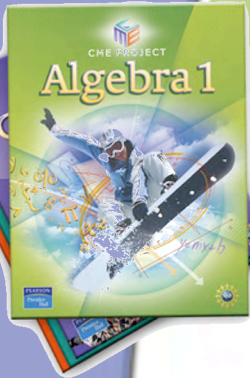
-  EDC's Center for Mathematics Education
-  National Advisory Board
-  Core Mathematical Consultants
-  Teacher Advisory Board
-  Field-Test Teachers



CME Project Overview

Fundamental Organizing Principle

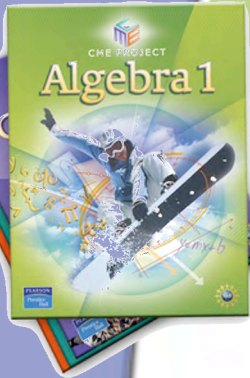
The widespread utility and effectiveness of mathematics come not just from mastering specific skills, topics, and techniques, but more importantly, from developing the ways of thinking—the habits of mind—used to create the results.



CME Project Overview

“Traditional” course structure: it’s familiar but different

- 📖 Structured around the sequence of Algebra 1, Geometry, Algebra 2, Precalculus
- 📖 Uses a variety of instructional approaches
- 📖 Focuses on particular mathematical habits
- 📖 Uses examples and contexts from many fields
- 📖 Organized around mathematical themes

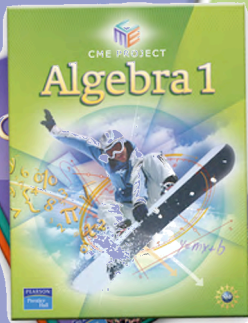


CME Project Overview

CME Project audience:

the (large number of) teachers who...

- Want the familiar course structure
- Want a problem- and exploration-based program
- Want to bring activities to “closure”
- Want rigor and accessibility for all






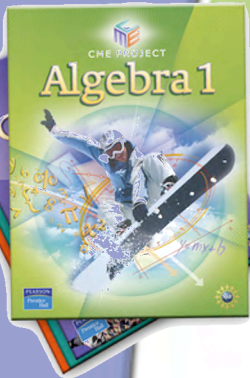
CME Project Overview

Relationship with Texas Instruments

CME Project makes essential use of technology:



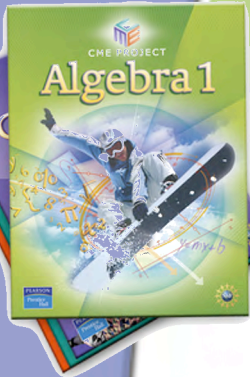
-  A “function-modeling” language (FML)
-  A computer algebra system (CAS)
-  An interactive geometry environment



CME Project Overview

Why CAS-Based Technology?

- ❏ To make tractable and to enhance many beautiful classical topics, historically considered too technical for high school students, by ***reducing computational overhead***.
- ❏ To provide students a platform for ***experimenting*** with algebraic expressions and other mathematical objects in the same way that calculators can be used to experiment with numbers.
- ❏ To allow students to build computational models of algebraic objects that have no faithful physical counterparts, ***highlighting similarities in algebraic structures***.

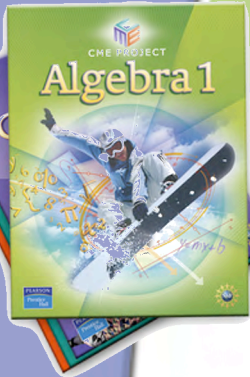


Reducing Computational Overhead

Lagrange Interpolation

- Find a polynomial function that fits this table of data.

1	-5
2	1
5	-29



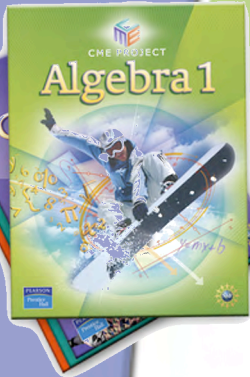
Reducing Computational Overhead

Lagrange Interpolation

 Write the polynomial in this form.

$$f(x) = A(x - 2)(x - 5) + B(x - 1)(x - 5) + C(x - 1)(x - 2)$$

1	-5
2	1
5	-29



Reducing Computational Overhead

Lagrange Interpolation

$$f(x) = A(x - 2)(x - 5) + B(x - 1)(x - 5) + C(x - 1)(x - 2)$$

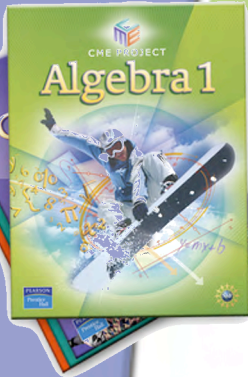
For $f(1)$, the second two terms drop out, so

$$f(1) = A(1 - 2)(1 - 5)$$

$$-5 = 4A$$

$$A = -\frac{5}{4}$$

1	-5
2	1
5	-29



Reducing Computational Overhead

Lagrange Interpolation

$$f(x) = A(x - 2)(x - 5) + B(x - 1)(x - 5) + C(x - 1)(x - 2)$$

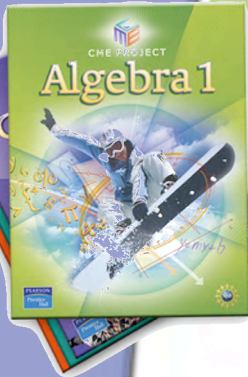
For $f(2)$, the first and last terms drop out, so

$$f(2) = B(2 - 1)(2 - 5)$$

$$1 = -3B$$

$$B = -\frac{1}{3}$$

1	-5
2	1
5	-29



Reducing Computational Overhead

Lagrange Interpolation

$$f(x) = A(x - 2)(x - 5) + B(x - 1)(x - 5) + C(x - 1)(x - 2)$$

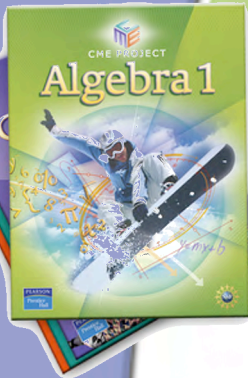
For $f(5)$, the last two terms drop out, so

$$f(5) = C(5 - 1)(5 - 2)$$

$$-29 = 12C$$

$$C = -\frac{29}{12}$$

1	-5
2	1
5	-29



Reducing Computational Overhead

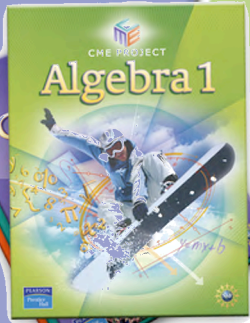
Lagrange Interpolation

So the following function fits the table.

$$f(x) = \left(-\frac{5}{4}\right)(x-2)(x-5) + \left(-\frac{1}{3}\right)(x-1)(x-5) + \left(-\frac{29}{12}\right)(x-1)(x-2)$$

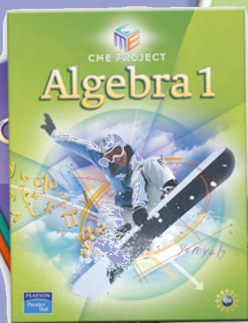
Now, write the function in normal form.

1	-5
2	1
5	-29



Experimenting

A New Factor Game for Precalculus



Experimenting

1.1 Playing the Factor Game

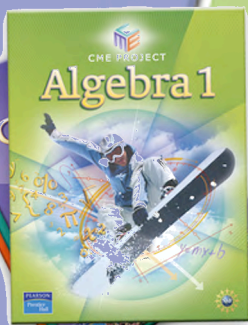
Playing the Factor Game is a fun way to practice finding factors of whole numbers. If you pay close attention, you may learn some interesting things about numbers that you didn't know before! To play the game, you need a Factor Game Board and colored pens, pencils, or markers.

active math
online

For: Factor Game Activity
Visit: PMSchool.com
Web Code: amd-1101

The Factor Game

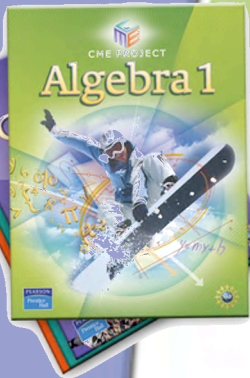
1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30



Experimenting

The Polynomial Factor Game

$x - 1$	$x^2 - 1$	$x^3 - 1$	$x^4 - 1$	$x^5 - 1$
$x^6 - 1$	$x^7 - 1$	$x^8 - 1$	$x^9 - 1$	$x^{10} - 1$
$x^{11} - 1$	$x^{12} - 1$	$x^{13} - 1$	$x^{14} - 1$	$x^{15} - 1$
$x^{16} - 1$	$x^{17} - 1$	$x^{18} - 1$	$x^{19} - 1$	$x^{20} - 1$
$x^{21} - 1$	$x^{22} - 1$	$x^{23} - 1$	$x^{24} - 1$	$x^{25} - 1$
$x^{26} - 1$	$x^{27} - 1$	$x^{28} - 1$	$x^{29} - 1$	$x^{30} - 1$



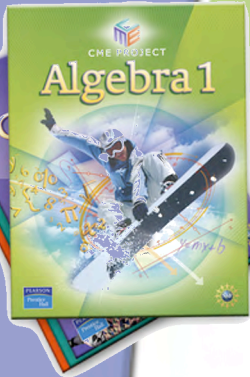
Highlighting Similarities in Algebraic Structures

Find an integer whose remainder is

 2 when divided by 3,

 4 when divided by 5, and

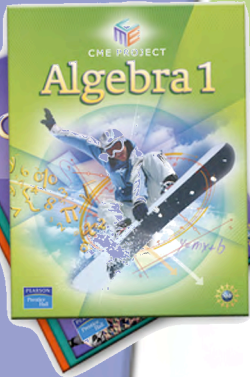
 3 when divided by 7.



Highlighting Similarities in Algebraic Structures

Write the number as

$$A(5 \cdot 7) + B(3 \cdot 7) + C(3 \cdot 5)$$

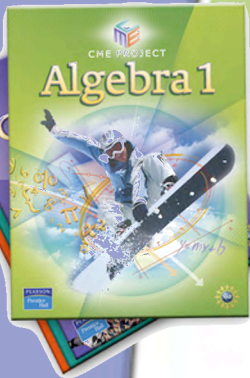


Highlighting Similarities in Algebraic Structures

Write the number as

$$A(35) + B(21) + C(15)$$

- when divided by 3, the last two terms drop out.
- So $35A / 3$ has a remainder of 2.
- $A=1$ works, since $35 / 3$ has remainder 2.

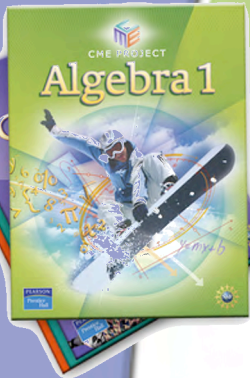


Highlighting Similarities in Algebraic Structures

Write the number as

$$A(35) + B(21) + C(15)$$

- when divided by 5, the first and last terms drop out.
- So $21B / 5$ has a remainder of 4.
- $21 / 5$ has a remainder of 1, so $B=4$ works, since $84 / 5$ has remainder 4.

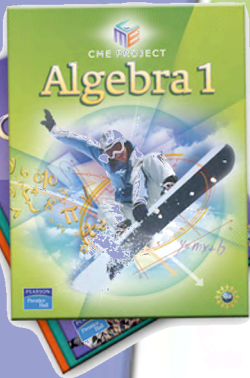


Highlighting Similarities in Algebraic Structures

Write the number as

$$A(35) + B(21) + C(15)$$

- when divided by 7, the first two terms drop out.
- So $15C / 7$ has a remainder of 3.
- $15 / 7$ has a remainder of 1, so $B=3$ works, since $45 / 7$ has remainder 3.

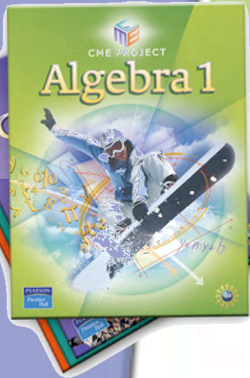


Highlighting Similarities in Algebraic Structures

So the number

$$1(35) + 4(21) + 3(15) = 164$$

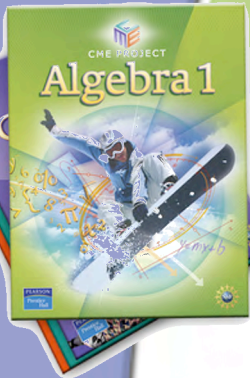
fits the pattern.



Highlighting Similarities in Algebraic Structures

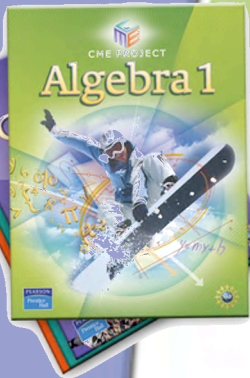
164 is an integer whose remainder is
2 when divided by 3,
4 when divided by 5, and
3 when divided by 7.

But it's not the only one. In fact, for
any integer k , $164 + 105k$ fits the
pattern.



Highlighting Similarities in Algebraic Structures

The process underneath the “Chinese Remainder Theorem” is similar to the concept underneath Lagrange Interpolation



Highlighting Similarities in Algebraic Structures

$f(x) = -4x^2 + 18x - 19$ is a polynomial function such that

$$f(1) = -5,$$

$$f(2) = 1, \text{ and}$$

$$f(5) = -29.$$

But it's not the only one. In fact, for any real number a ,

$$f(x) = -4x^2 + 18x - 19 + a(x - 1)(x - 2)(x - 5)$$

fits the pattern.

CME Project Availability Dates

Available right now!

- CME Project Algebra 1 Sampler
- CME Project Geometry Sampler

Available in Summer 2007

- CME Project Technology Sampler

Available in Fall 2007

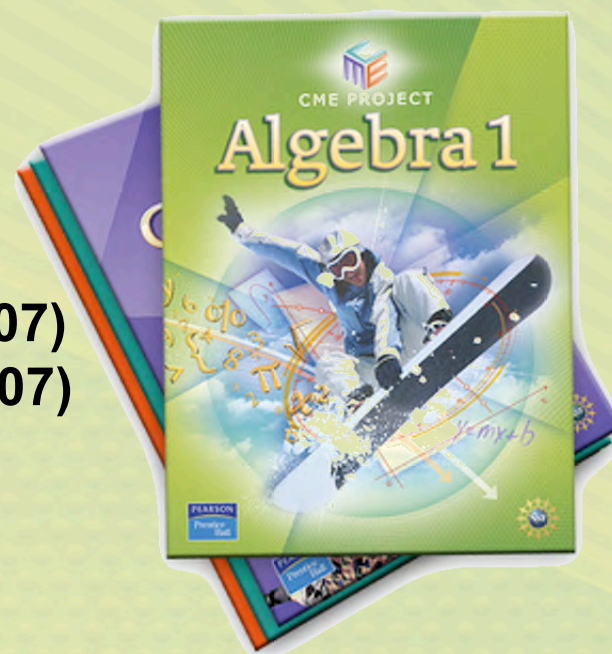
- CME Algebra 1 text (November 2007)
- CME Geometry text (November 2007)

Available in Spring 2008

- CME Algebra 2 text

Available in Summer 2008


- CME Precalculus text




CME Project

For more information

 www.edc.org/cmeproject

 www.phschool.org/cme

 **Al Cuoco**
acuoco@edc.org

 **Kevin Waterman**
kwaterman@edc.org

