Algebra for Teaching

Some Recommendations for Teacher Preparation

Al Cuoco

Center for Mathematics Education
EDC
acuoco@edc.org

MSRI Critical Issues, 2008

Outline

1. In a Typical Day...
   - Algebra in algebra class
   - Algebra in non-algebra classes
   - Algebra outside of class
2. Some Recommendations
   - For Abstract Algebra
   - For Linear Algebra
   - For Number Theory
   - For Other Things
3. Conclusion

In algebra class

- Is $\frac{x^2-3x}{x^2-9}$ the same as $\frac{x}{x+3}$?

IN ALGEBRA CLASS

- Is $\frac{x^2-3x}{x^2-9}$ the same as $\frac{x}{x+3}$?
- When is $x^n - 1$ a factor of $x^m - 1$?

In algebra class

- Is $\frac{x^2-3x}{x^2-9}$ the same as $\frac{x}{x+3}$?
- When is $x^n - 1$ a factor of $x^m - 1$?
- Can a quadratic equation have more than 2 roots?

In algebra class

- Is $\frac{x^2-3x}{x^2-9}$ the same as $\frac{x}{x+3}$?
- When is $x^n - 1$ a factor of $x^m - 1$?
- Can a quadratic equation have more than 2 roots?
- At how many values do two polynomials have to agree before we can say they are equal?

In algebra class

- Is $\frac{x^2-3x}{x^2-9}$ the same as $\frac{x}{x+3}$?
- When is $x^n - 1$ a factor of $x^m - 1$?
- Can a quadratic equation have more than 2 roots?
- At how many values do two polynomials have to agree before we can say they are equal?
- And what does it *mean* for two polynomials to be equal?

IN ALGEBRA CLASS

- Is $\frac{x^2-3x}{x^2-9}$ the same as $\frac{x}{x+3}$?
- When is $x^n - 1$ a factor of $x^m - 1$?
- Can a quadratic equation have more than 2 roots?
- At how many values do two polynomials have to agree before we can say they are equal?
- And what does it *mean* for two polynomials to be equal?
- Can a system of linear equations with integer coefficients have an irrational solution?

IN ALGEBRA CLASS

- Is $\frac{x^2-3x}{x^2-9}$ the same as $\frac{x}{x+3}$?
- When is $x^n - 1$ a factor of $x^m - 1$?
- Can a quadratic equation have more than 2 roots?
- At how many values do two polynomials have to agree before we can say they are equal?
- And what does it *mean* for two polynomials to be equal?
- Can a system of linear equations with integer coefficients have an irrational solution?
- Can a system of 3 linear equations in four unknowns have exactly 3 solutions?

In algebra class

- Is $\frac{x^2-3x}{x^2-9}$ the same as $\frac{x}{x+3}$?
- When is $x^n - 1$ a factor of $x^m - 1$?
- Can a quadratic equation have more than 2 roots?
- At how many values do two polynomials have to agree before we can say they are equal?
- And what does it *mean* for two polynomials to be equal?
- Can a system of linear equations with integer coefficients have an irrational solution?
- Can a system of 3 linear equations in four unknowns have exactly 3 solutions?
- If a polynomial doesn't factor over $\mathbb{Z}$, can it factor over $\mathbb{Q}$? (McCallum)

IN OTHER COURSES

- Why is arithmetic with complex numbers like arithmetic with polynomials?

IN OTHER COURSES

- Why is arithmetic with complex numbers like arithmetic with polynomials?
- What does it mean for two functions to be equal?

IN OTHER COURSES

- Why is arithmetic with complex numbers like arithmetic with polynomials?
- What does it mean for two functions to be equal?
- How can I find a polynomial that agrees with a table?

In other courses

- Why is arithmetic with complex numbers like arithmetic with polynomials?
- What does it mean for two functions to be equal?
- How can I find a polynomial that agrees with a table?
- Why do they use f^{-1} for inverse function?

IN OTHER COURSES

- Why is arithmetic with complex numbers like arithmetic with polynomials?
- What does it mean for two functions to be equal?
- How can I find a polynomial that agrees with a table?
- Why do they use f^{-1} for inverse function?
- For polynomials, why does the h in the denominator of

$$\frac{f(x+h)-f(x)}{h}$$

always cancel out?

IN OTHER COURSES

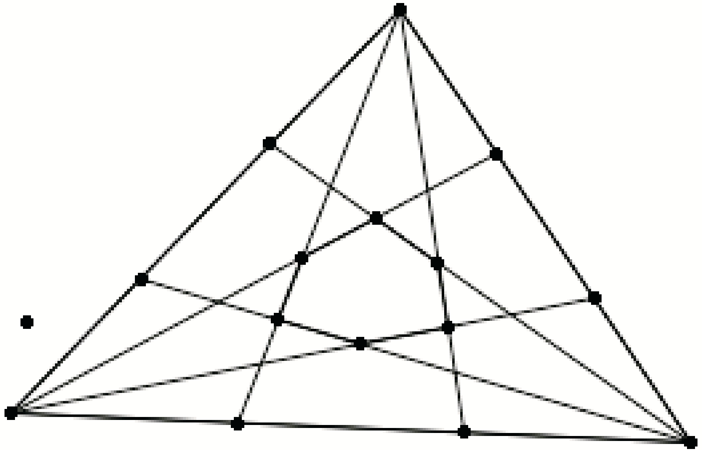
- Why is arithmetic with complex numbers like arithmetic with polynomials?
- What does it mean for two functions to be equal?
- How can I find a polynomial that agrees with a table?
- Why do they use f^{-1} for inverse function?
- For polynomials, why does the h in the denominator of

$$\frac{f(x+h)-f(x)}{h}$$

always cancel out?

- *Why* can't you trisect a 60° angle with a straightedge and compass?

Geo



5.01388 cm^2

50.1387 cm^2

Professional uses of algebra

The class is using calculators and estimation to get decimal approximations to $\sqrt{5}$. One student looks at how you do out long multiplication and realizes that none of these decimals would ever work, because if you square a finite (non-integer) decimal, there'll be a digit to the right of the decimal point. So you can't ever get an integer. She deduces that that $\sqrt{5}$ can't be rational.

— adapted from "A Dialogue About Teaching" in *What's Happening in Math Class?* Teacher's College Press.

PROFESSIONAL USES OF ALGEBRA

Nine year old David, experimenting with numbers, conjectures that, if the period for the decimal expansion of $\frac{1}{n}$ is $n - 1$, then n is prime.

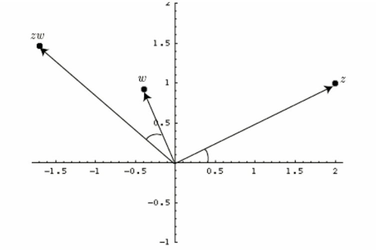
— Adapted from a Reader Reflection by Walt Levisee in the *Mathematics Teacher* (March, 1997).

Speaking of decimals, how would you characterize the "unit fractions" $\frac{1}{n}$ that have terminating decimal expansions? What can you say about the periods of the repeating ones?

Professional uses of algebra

How can you help your students understand the "multiplication rule" for complex numbers?

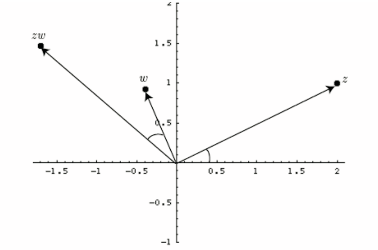
$$|zw| = |z|\,|w| \text{ and } \mathrm{Arg}(zw) = \mathrm{Arg}(z) + \mathrm{Arg}(w)$$



PROFESSIONAL USES OF ALGEBRA

How can you help your students understand the "multiplication rule" for complex numbers?

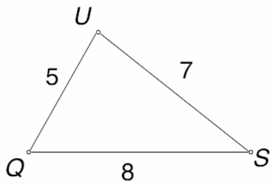
$$|zw| = |z|\,|w| \text{ and } \mathrm{Arg}(zw) = \mathrm{Arg}(z) + \mathrm{Arg}(w)$$



What if they don't know any trig?

PROFESSIONAL USES OF ALGEBRA

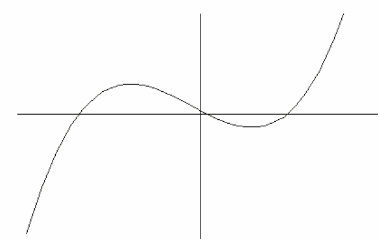
How can you generate “nice” problems, for example:



How big is $\angle Q$?

PROFESSIONAL USES OF ALGEBRA

Or...



$$f(x) = 140 - 144\,x + 3\,x^2 + x^3$$

Find the zeros, extrema, and inflection points

Suggestions for abstract algebra

Instill a sense that algebraic objects are open to experiment

SUGGESTIONS FOR ABSTRACT ALGEBRA

Instill a sense that algebraic objects are open to experiment

- Start with rings, fields, and polynomials—not groups

Suggestions for Abstract Algebra

Instill a sense that algebraic objects are open to experiment

- Start with rings, fields, and polynomials—not groups
- Solve equations in these systems

SUGGESTIONS FOR ABSTRACT ALGEBRA

Instill a sense that algebraic objects are open to experiment

- Start with rings, fields, and polynomials—not groups
- Solve equations in these systems
- Develop in detail the structural similarities and differences between $\mathbb{Z}$ and $\mathbb{Q}[x]$

SUGGESTIONS FOR ABSTRACT ALGEBRA

Instill a sense that algebraic objects are open to experiment

- Start with rings, fields, and polynomials—not groups
- Solve equations in these systems
- Develop in detail the structural similarities and differences between $\mathbb{Z}$ and $\mathbb{Q}[x]$
- Construct $\mathbb{C}$ as a $\mathbb{R}[x]/(x^2+1)\mathbb{R}[x]$

SUGGESTIONS FOR ABSTRACT ALGEBRA

Instill a sense that algebraic objects are open to experiment

- Start with rings, fields, and polynomials—not groups
- Solve equations in these systems
- Develop in detail the structural similarities and differences between $\mathbb{Z}$ and $\mathbb{Q}[x]$
- Construct $\mathbb{C}$ as a $\mathbb{R}[x]/(x^2+1)\mathbb{R}[x]$
- Construct more general splitting fields in the same way

Suggestions for Abstract Algebra

Instill a sense that algebraic objects are open to experiment

- Start with rings, fields, and polynomials—not groups
- Solve equations in these systems
- Develop in detail the structural similarities and differences between $\mathbb{Z}$ and $\mathbb{Q}[x]$
- Construct $\mathbb{C}$ as a $\mathbb{R}[x]/(x^2+1)\mathbb{R}[x]$
- Construct more general splitting fields in the same way
- Tie Galois theory to ...

Suggestions for Abstract Algebra

Instill a sense that algebraic objects are open to experiment

- Start with rings, fields, and polynomials—not groups
- Solve equations in these systems
- Develop in detail the structural similarities and differences between $\mathbb{Z}$ and $\mathbb{Q}[x]$
- Construct $\mathbb{C}$ as a $\mathbb{R}[x]/(x^2+1)\mathbb{R}[x]$
- Construct more general splitting fields in the same way
- Tie Galois theory to ...
  - the theory of equations

Suggestions for Abstract Algebra

Instill a sense that algebraic objects are open to experiment

- Start with rings, fields, and polynomials—not groups
- Solve equations in these systems
- Develop in detail the structural similarities and differences between $\mathbb{Z}$ and $\mathbb{Q}[x]$
- Construct $\mathbb{C}$ as a $\mathbb{R}[x]/(x^2+1)\mathbb{R}[x]$
- Construct more general splitting fields in the same way
- Tie Galois theory to . . .
  - the theory of equations
  - roots of unity

Suggestions for Abstract Algebra

Instill a sense that algebraic objects are open to experiment

- Start with rings, fields, and polynomials—not groups
- Solve equations in these systems
- Develop in detail the structural similarities and differences between $\mathbb{Z}$ and $\mathbb{Q}[x]$
- Construct $\mathbb{C}$ as a $\mathbb{R}[x]/(x^2+1)\mathbb{R}[x]$
- Construct more general splitting fields in the same way
- Tie Galois theory to ...
  - the theory of equations
  - roots of unity
  - the geometry of the regular n-gon

Suggestions for Abstract Algebra

Instill a sense that algebraic objects are open to experiment

- Start with rings, fields, and polynomials—not groups
- Solve equations in these systems
- Develop in detail the structural similarities and differences between $\mathbb{Z}$ and $\mathbb{Q}[x]$
- Construct $\mathbb{C}$ as a $\mathbb{R}[x]/(x^2+1)\mathbb{R}[x]$
- Construct more general splitting fields in the same way
- Tie Galois theory to ...
  - the theory of equations
  - roots of unity
  - the geometry of the regular n-gon
  - Cardano

Suggestions for Abstract Algebra

Instill a sense that algebraic objects are open to experiment

- Start with rings, fields, and polynomials—not groups
- Solve equations in these systems
- Develop in detail the structural similarities and differences between $\mathbb{Z}$ and $\mathbb{Q}[x]$
- Construct $\mathbb{C}$ as a $\mathbb{R}[x]/(x^2 + 1)\mathbb{R}[x]$
- Construct more general splitting fields in the same way
- Tie Galois theory to . . .
  - the theory of equations
  - roots of unity
  - the geometry of the regular n-gon
  - Cardano
  - the theory of SE and compass constructibility

SUGGESTIONS FOR LINEAR ALGEBRA

Help students develop a knack for reasoning by linearity

SUGGESTIONS FOR LINEAR ALGEBRA

Help students develop a knack for reasoning by linearity

$$(*)\begin{cases} 3x + 5y - 7z + w & = 2 \\ 4x - 5y + 10z - w & = 8 \\ 6x - 5y + 3z + 2w & = 6 \end{cases}$$

SUGGESTIONS FOR LINEAR ALGEBRA

- Stress some basics. . .

SUGGESTIONS FOR LINEAR ALGEBRA

- Stress some basics. . .
  - extension by linearity

SUGGESTIONS FOR LINEAR ALGEBRA

- Stress some basics. . .
  - extension by linearity
  - change of basis

SUGGESTIONS FOR LINEAR ALGEBRA

- Stress some basics. . .
  - extension by linearity
  - change of basis
- Carefully develop vectorial methods in geometry. . .

Suggestions for linear algebra

- Stress some basics...
  - extension by linearity
  - change of basis
- Carefully develop vectorial methods in geometry...
  - including the "extension program" from $\mathbb{R}^{2 \text{ or } 3}$ to $\mathbb{R}^n$...

Suggestions for linear algebra

- Stress some basics...
  - extension by linearity
  - change of basis
- Carefully develop vectorial methods in geometry...
  - including the "extension program" from $\mathbb{R}^{2 \text{ or } 3}$ to $\mathbb{R}^n$...
  - and the use of matrices to represent linear transformations

Suggestions for linear algebra

- Stress some basics. . .
  - extension by linearity
  - change of basis
- Carefully develop vectorial methods in geometry. . .
  - including the "extension program" from $\mathbb{R}^{2 \text{ or } 3}$ to $\mathbb{R}^n$. . .
  - and the use of matrices to represent linear transformations
- Connect various uses of determinants. . .

Suggestions for linear algebra

- Stress some basics...
  - extension by linearity
  - change of basis
- Carefully develop vectorial methods in geometry...
  - including the "extension program" from $\mathbb{R}^{2 \text{ or } 3}$ to $\mathbb{R}^n$...
  - and the use of matrices to represent linear transformations
- Connect various uses of determinants...
  - as area and volume

Suggestions for linear algebra

- Stress some basics...
  - extension by linearity
  - change of basis
- Carefully develop vectorial methods in geometry...
  - including the “extension program” from $\mathbb{R}^{2 \text{ or } 3}$ to $\mathbb{R}^n$...
  - and the use of matrices to represent linear transformations
- Connect various uses of determinants...
  - as area and volume
  - as a test for linear dependence

Suggestions for linear algebra

- Stress some basics...
  - extension by linearity
  - change of basis
- Carefully develop vectorial methods in geometry...
  - including the "extension program" from $\mathbb{R}^{2 \text{ or } 3}$ to $\mathbb{R}^n$...
  - and the use of matrices to represent linear transformations
- Connect various uses of determinants...
  - as area and volume
  - as a test for linear dependence
  - as an algebraic tool (resultants, Cramer, matrix inverses...)

Suggestions for linear algebra

- Stress some basics...
  - extension by linearity
  - change of basis
- Carefully develop vectorial methods in geometry...
  - including the "extension program" from $\mathbb{R}^{2 \text{ or } 3}$ to $\mathbb{R}^n$...
  - and the use of matrices to represent linear transformations
- Connect various uses of determinants...
  - as area and volume
  - as a test for linear dependence
  - as an algebraic tool (resultants, Cramer, matrix inverses...)
- Exploit matrix algebra as a formal "bookkeeping" tool...

Suggestions for linear algebra

- Stress some basics. . .
  - extension by linearity
  - change of basis
- Carefully develop vectorial methods in geometry. . .
  - including the "extension program" from $\mathbb{R}^{2 \text{ or } 3}$ to $\mathbb{R}^n$. . .
  - and the use of matrices to represent linear transformations
- Connect various uses of determinants. . .
  - as area and volume
  - as a test for linear dependence
  - as an algebraic tool (resultants, Cramer, matrix inverses. . .)
- Exploit matrix algebra as a formal "bookkeeping" tool. . .
  - in adjacency and scheduling problems

Suggestions for linear algebra

- Stress some basics...
  - extension by linearity
  - change of basis
- Carefully develop vectorial methods in geometry...
  - including the "extension program" from $\mathbb{R}^{2 \text{ or } 3}$ to $\mathbb{R}^n$...
  - and the use of matrices to represent linear transformations
- Connect various uses of determinants...
  - as area and volume
  - as a test for linear dependence
  - as an algebraic tool (resultants, Cramer, matrix inverses...)
- Exploit matrix algebra as a formal "bookkeeping" tool...
  - in adjacency and scheduling problems
  - as tools for solving recurrence equations

Suggestions for linear algebra

- Stress some basics...
  - extension by linearity
  - change of basis
- Carefully develop vectorial methods in geometry...
  - including the "extension program" from $\mathbb{R}^{2 \text{ or } 3}$ to $\mathbb{R}^n$...
  - and the use of matrices to represent linear transformations
- Connect various uses of determinants...
  - as area and volume
  - as a test for linear dependence
  - as an algebraic tool (resultants, Cramer, matrix inverses...)
- Exploit matrix algebra as a formal "bookkeeping" tool...
  - in adjacency and scheduling problems
  - as tools for solving recurrence equations
- Make connections among eigenvalues, geometry, and algebra

Suggestions for number theory

Let general results evolve from numerical experiments

Suggestions for number theory

Let general results evolve from numerical experiments

- Compare arithmetic in $\mathbb{Z}$, $\mathbb{Z}/n\mathbb{Z}$, $\mathbb{Z}[i]$, and $\mathbb{Q}[x]$

SUGGESTIONS FOR NUMBER THEORY

Let general results evolve from numerical experiments

- Compare arithmetic in $\mathbb{Z}$, $\mathbb{Z}/n\mathbb{Z}$, $\mathbb{Z}[i]$, and $\mathbb{Q}[x]$
- Examine Euclid's algorithm in $\mathbb{Z}$, $\mathbb{Z}[i]$, and $\mathbb{Q}[x]$

SUGGESTIONS FOR NUMBER THEORY

Let general results evolve from numerical experiments

- Compare arithmetic in $\mathbb{Z}$, $\mathbb{Z}/n\mathbb{Z}$, $\mathbb{Z}[i]$, and $\mathbb{Q}[x]$
- Examine Euclid's algorithm in $\mathbb{Z}$, $\mathbb{Z}[i]$, and $\mathbb{Q}[x]$
- Make localization and reduction general purpose tools

Suggestions for Number Theory

Let general results evolve from numerical experiments

- Compare arithmetic in $\mathbb{Z}$, $\mathbb{Z}/n\mathbb{Z}$, $\mathbb{Z}[i]$, and $\mathbb{Q}[x]$
- Examine Euclid's algorithm in $\mathbb{Z}$, $\mathbb{Z}[i]$, and $\mathbb{Q}[x]$
- Make localization and reduction general purpose tools
- Connect Pythagorean and Eisenstein triples to. . .

Suggestions for number theory

Let general results evolve from numerical experiments

- Compare arithmetic in $\mathbb{Z}$, $\mathbb{Z}/n\mathbb{Z}$, $\mathbb{Z}[i]$, and $\mathbb{Q}[x]$
- Examine Euclid's algorithm in $\mathbb{Z}$, $\mathbb{Z}[i]$, and $\mathbb{Q}[x]$
- Make localization and reduction general purpose tools
- Connect Pythagorean and Eisenstein triples to...
  - norms from quadratic fields

Suggestions for Number Theory

Let general results evolve from numerical experiments

- Compare arithmetic in $\mathbb{Z}$, $\mathbb{Z}/n\mathbb{Z}$, $\mathbb{Z}[i]$, and $\mathbb{Q}[x]$
- Examine Euclid's algorithm in $\mathbb{Z}$, $\mathbb{Z}[i]$, and $\mathbb{Q}[x]$
- Make localization and reduction general purpose tools
- Connect Pythagorean and Eisenstein triples to...
  - norms from quadratic fields a
  - rational points on conics

SUGGESTIONS FOR NUMBER THEORY

Let general results evolve from numerical experiments

- Compare arithmetic in $\mathbb{Z}$, $\mathbb{Z}/n\mathbb{Z}$, $\mathbb{Z}[i]$, and $\mathbb{Q}[x]$
- Examine Euclid's algorithm in $\mathbb{Z}$, $\mathbb{Z}[i]$, and $\mathbb{Q}[x]$
- Make localization and reduction general purpose tools
- Connect Pythagorean and Eisenstein triples to. . .
  - norms from quadratic fields a
  - rational points on conics
- Develop the theory of repeating decimals

SUGGESTIONS FOR NUMBER THEORY

Let general results evolve from numerical experiments

- Compare arithmetic in $\mathbb{Z}$, $\mathbb{Z}/n\mathbb{Z}$, $\mathbb{Z}[i]$, and $\mathbb{Q}[x]$
- Examine Euclid's algorithm in $\mathbb{Z}$, $\mathbb{Z}[i]$, and $\mathbb{Q}[x]$
- Make localization and reduction general purpose tools
- Connect Pythagorean and Eisenstein triples to...
  - norms from quadratic fields a
  - rational points on conics
- Develop the theory of repeating decimals
- Connect the Chinese remainder theorem to Lagrange interpolation

Suggestions for Number Theory

Let general results evolve from numerical experiments

- Compare arithmetic in $\mathbb{Z}$, $\mathbb{Z}/n\mathbb{Z}$, $\mathbb{Z}[i]$, and $\mathbb{Q}[x]$
- Examine Euclid's algorithm in $\mathbb{Z}$, $\mathbb{Z}[i]$, and $\mathbb{Q}[x]$
- Make localization and reduction general purpose tools
- Connect Pythagorean and Eisenstein triples to...
  - norms from quadratic fields a
  - rational points on conics
- Develop the theory of repeating decimals
- Connect the Chinese remainder theorem to Lagrange interpolation
- Talk to Glenn Stevens

Suggestions for other things

Encourage reasoning about calculations and operations

Suggestions for other things

Encourage reasoning about calculations and operations

- The theory of finite differences is useful in teaching

SUGGESTIONS FOR OTHER THINGS

Encourage reasoning about calculations and operations

- The theory of finite differences is useful in teaching
- Polynomial calculus is all algebra

Suggestions for other things

Encourage reasoning about calculations and operations

- The theory of finite differences is useful in teaching
- Polynomial calculus is all algebra
    - The remainder when $f(x)$ is divided by $(x-a)^2$

SUGGESTIONS FOR OTHER THINGS

Encourage reasoning about calculations and operations

- The theory of finite differences is useful in teaching
- Polynomial calculus is all algebra
  - The remainder when $f(x)$ is divided by $(x-a)^2$
  - The Taylor expansion about $x = a$ is a *polynomial identity*

Suggestions for other things

Encourage reasoning about calculations and operations

- The theory of finite differences is useful in teaching
- Polynomial calculus is all algebra
  - The remainder when $f(x)$ is divided by $(x-a)^2$
  - The Taylor expansion about $x=a$ is a *polynomial identity*
- $\mathbb{C}$ can be used to derive the addition formulas for sine and cosine (rather than the other way around)

Suggestions for other things

Encourage reasoning about calculations and operations

- The theory of finite differences is useful in teaching
- Polynomial calculus is all algebra
  - The remainder when $f(x)$ is divided by $(x-a)^2$
  - The Taylor expansion about $x=a$ is a *polynomial identity*
- $\mathbb{C}$ can be used to derive the addition formulas for sine and cosine (rather than the other way around)
- Summatory polynomials are useful in both calculus and polynomial interpolation

Suggestions for other things

Encourage reasoning about calculations and operations

- The theory of finite differences is useful in teaching
- Polynomial calculus is all algebra
  - The remainder when $f(x)$ is divided by $(x-a)^2$
  - The Taylor expansion about $x=a$ is a *polynomial identity*
- $\mathbb{C}$ can be used to derive the addition formulas for sine and cosine (rather than the other way around)
- Summatory polynomials are useful in both calculus and polynomial interpolation
- Function algebra gives an example of algebraic structure

SUGGESTIONS FOR OTHER THINGS

Encourage reasoning about calculations and operations

- The theory of finite differences is useful in teaching
- Polynomial calculus is all algebra
  - The remainder when $f(x)$ is divided by $(x - a)^2$
  - The Taylor expansion about $x = a$ is a *polynomial identity*
- $\mathbb{C}$ can be used to derive the addition formulas for sine and cosine (rather than the other way around)
- Summatory polynomials are useful in both calculus and polynomial interpolation
- Function algebra gives an example of algebraic structure
- Statistics has deep connections to linear algebra

SUGGESTIONS FOR OTHER THINGS

Encourage reasoning about calculations and operations

- The theory of finite differences is useful in teaching
- Polynomial calculus is all algebra
  - The remainder when $f(x)$ is divided by $(x - a)^2$
  - The Taylor expansion about $x = a$ is a *polynomial identity*
- $\mathbb{C}$ can be used to derive the addition formulas for sine and cosine (rather than the other way around)
- Summatory polynomials are useful in both calculus and polynomial interpolation
- Function algebra gives an example of algebraic structure
- Statistics has deep connections to linear algebra
- Chebyshev polynomials bring coherence to trig addition formulas

IN CONCLUSION...

Algebra and algebraic reasoning around topics in the undergraduate mathematics curriculum can help prospective teachers enter the profession with a coherent view of secondary mathematics.

IN CONCLUSION...

Algebra and algebraic reasoning around topics in the undergraduate mathematics curriculum can help prospective teachers enter the profession with a coherent view of secondary mathematics.

But this doesn't come for free. Explicit connections to the daily work of high school teaching should be a part of every undergraduate course.

In conclusion. . .

This doesn't mean developing courses in high school mathematics from an "advanced" perspective. It means developing courses that develop the content and methods of undergraduate mathematics while taking seriously the profession-specific needs of high school teachers.

In conclusion...

This doesn't mean developing courses in high school mathematics from an "advanced" perspective. It means developing courses that develop the content and methods of undergraduate mathematics while taking seriously the profession-specific needs of high school teachers.

Conjecture: Such courses would benefit *all* mathematics majors, not just prospective teachers.

THANKS

Al Cuoco (acuoco@edc.org)

www.edc.org/cme