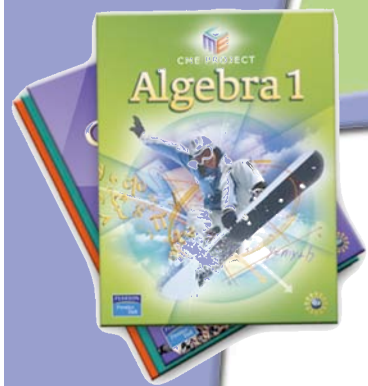
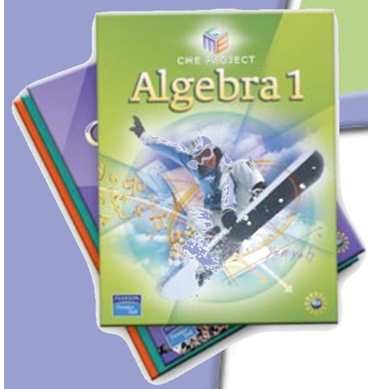


Curriculum and Technology: Evolving Together

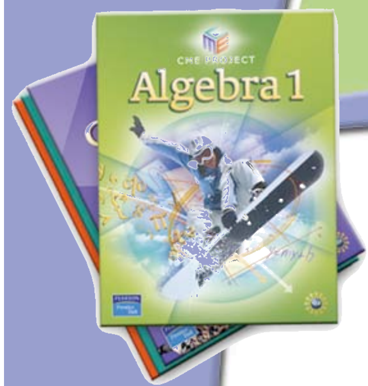
Kevin Waterman

Education Development Center



Experimenting

A New Factor Game for Precalculus



Experimenting

1.1 Playing the Factor Game

Playing the Factor Game is a fun way to practice finding factors of whole numbers. If you pay close attention, you may learn some interesting things about numbers that you didn't know before! To play the game, you need a Factor Game Board and colored pens, pencils, or markers.

active math online
For: Factor Game Activity
Visit: PHSchool.com
Web Code: amd-1101

The Factor Game

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30

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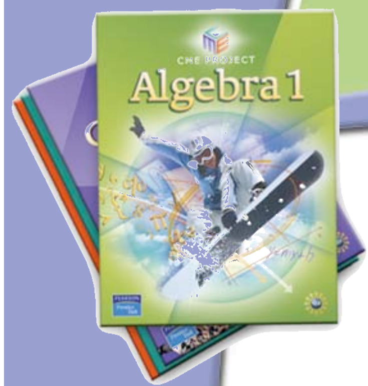
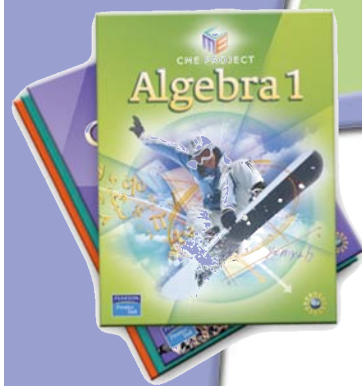


Experimenting

The Polynomial Factor Game

$x - 1$	$x^2 - 1$	$x^3 - 1$	$x^4 - 1$	$x^5 - 1$
$x^6 - 1$	$x^7 - 1$	$x^8 - 1$	$x^9 - 1$	$x^{10} - 1$
$x^{11} - 1$	$x^{12} - 1$	$x^{13} - 1$	$x^{14} - 1$	$x^{15} - 1$
$x^{16} - 1$	$x^{17} - 1$	$x^{18} - 1$	$x^{19} - 1$	$x^{20} - 1$
$x^{21} - 1$	$x^{22} - 1$	$x^{23} - 1$	$x^{24} - 1$	$x^{25} - 1$
$x^{26} - 1$	$x^{27} - 1$	$x^{28} - 1$	$x^{29} - 1$	$x^{30} - 1$

EDC

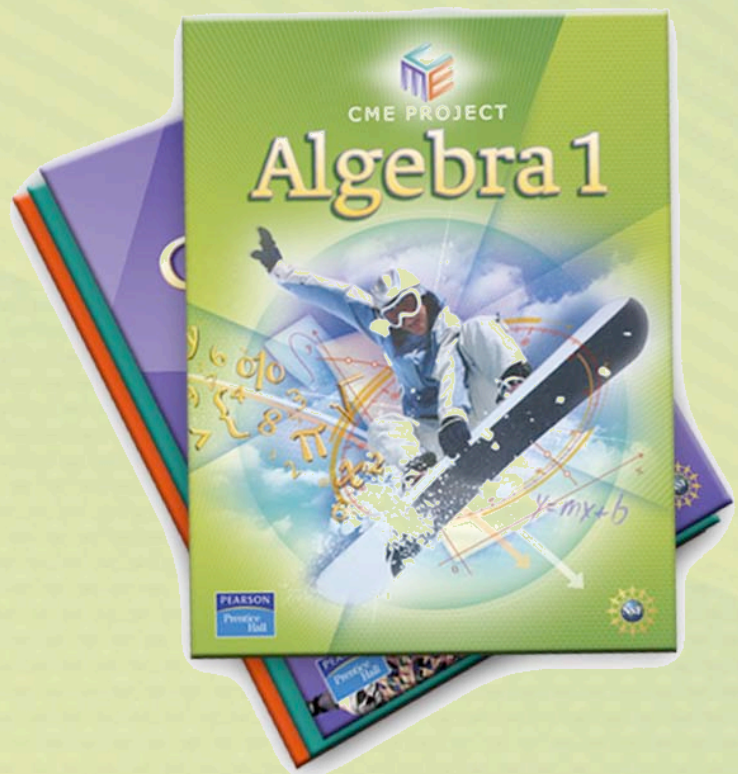


Curriculum and Technology: Evolving Together

- New technology features influence curriculum
- New curriculum ideas drive technology innovation

What is the CME Project?

- A Brand New, Comprehensive, 4-year Curriculum
- NSF-funded
- Problem-Based, Student-Centered Approach
- “Traditional” Course Structure



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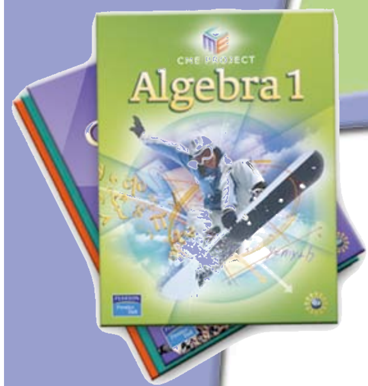
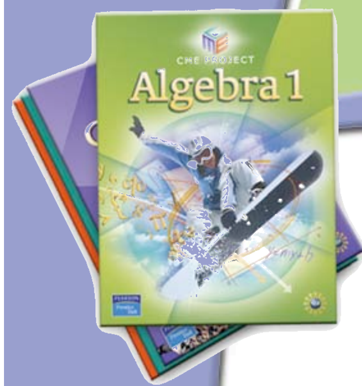


CME Project Overview

Contributors

- EDC's Center for Mathematics Education
- National Advisory Board
- Core Mathematical Consultants
- Teacher Advisory Board
- Field-Test Teachers



CME Project Overview

Fundamental Organizing Principle

The widespread utility and effectiveness of mathematics come not just from mastering specific skills, topics, and techniques, but more importantly, from developing the ways of thinking—the *habits of mind*—used to create the results.

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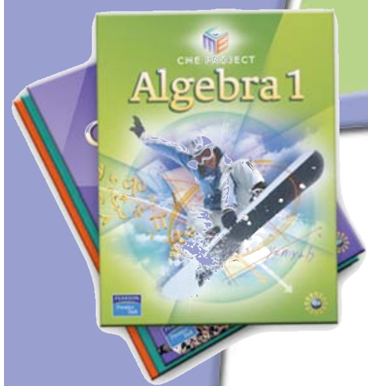

CME Project Overview

"Traditional" course structure: it's familiar but different

- Structured around the sequence of Algebra 1, Geometry, Algebra 2, Precalculus
- Uses a variety of instructional approaches
- Focuses on particular mathematical habits
- Uses examples and contexts from many fields
- Organized around mathematical themes

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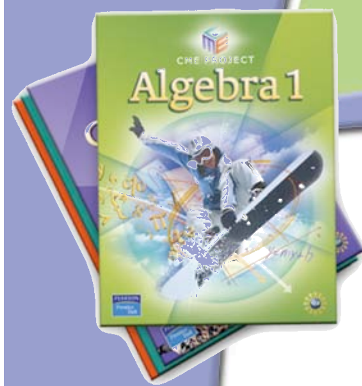

CME Project Overview

CME Project audience:

the (large number of) teachers who…

- Want the familiar course structure
- Want a problem- and exploration-based program
- Want to bring activities to “closure”
- Want rigor and accessibility for all

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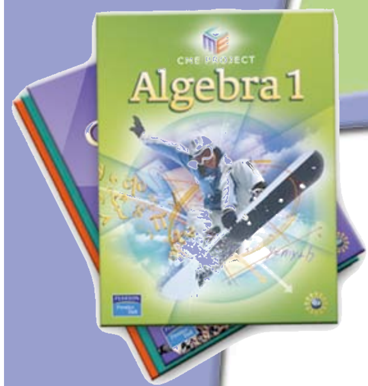
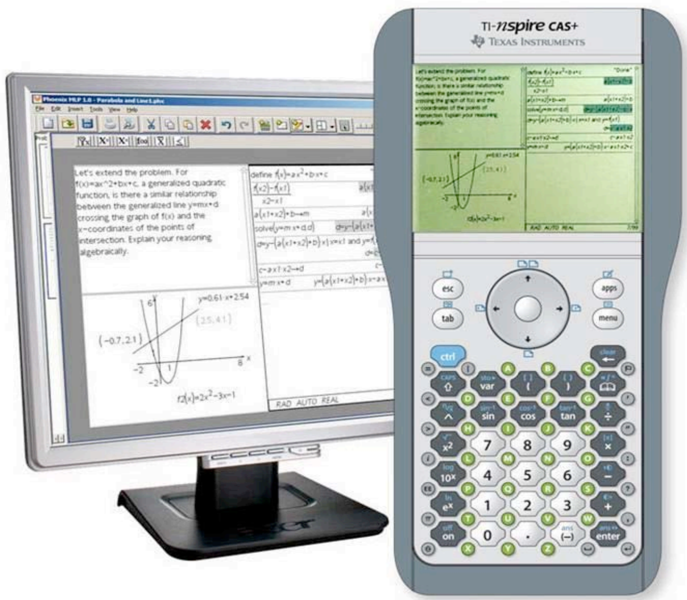


CME Project Overview

Relationship with Texas Instruments



CME Project makes essential use of technology:

- A “function-modeling” language (FML)
- A computer algebra system (CAS)
- An interactive geometry environment

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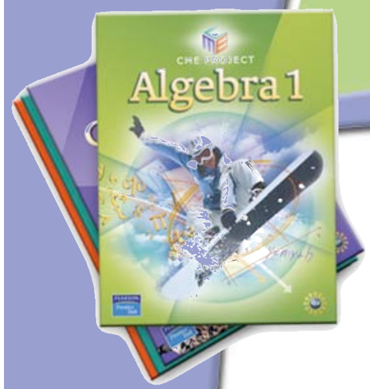

CME Project Overview

Why CAS-Based Technology?

- To provide students a platform for ***experimenting*** with algebraic expressions and other mathematical objects in the same way that calculators can be used to experiment with numbers.
- To make tractable and to enhance many beautiful classical topics, historically considered too technical for high school students, by ***reducing computational overhead***.
- To allow students to build computational models of algebraic objects that have no faithful physical counterparts, ***highlighting similarities in algebraic structures***.

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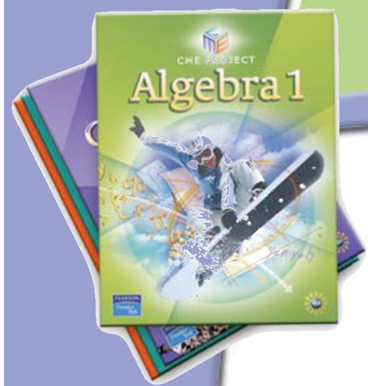
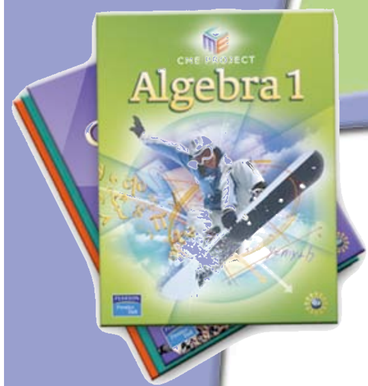


Experimenting

Defining Recursive Functions

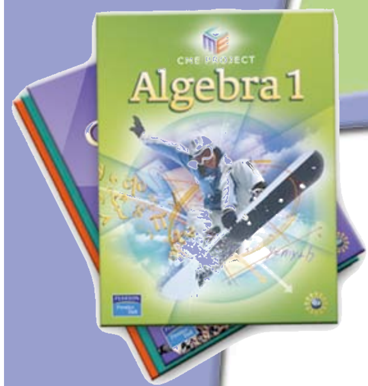
$$f(n) = \begin{cases} 3, & n = 0 \\ f(n-1) + 5, & n > 0 \end{cases}$$



Experimenting

Defining Recursive Functions on the TI-89

```
f(n)
Func
If n=0 Then
  Return 3
Else
  Return f(n-1) + 5
EndIf
EndFunc
```



Experimenting

Defining Recursive Functions on the TI-Nspire

- Entered in a template that looks just like what you write or see in a book
- Ease of creation prompted us to introduce recursive definitions in Algebra 1

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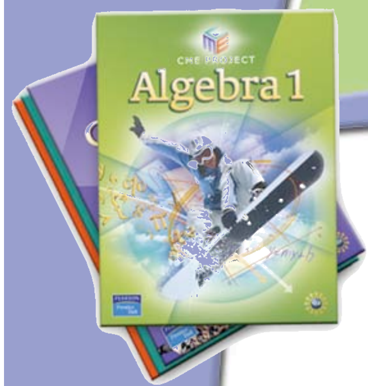


Experimenting

Estimate slope of a curve at a point:

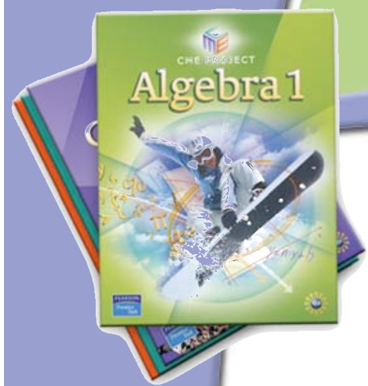
- On Graphing Calculators
  - Graph equation
  - Zoom in until graph “looks linear”
- On TI-Nspire Handheld
  - Graph equation
  - Construct two points and draw line
  - Move one point toward the other

Experimenting

Estimate the slope of

$$y = x^3 - x + 1$$

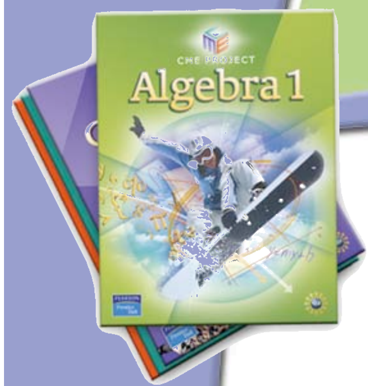
at point (1,1).



Linking Geometry to Algebra

You can determine the equation of the secant of $y = x^3 - x + 1$ through points $(1,1)$ and $(2,7)$ by dividing $x^3 - x + 1$ by $(x - 1)(x - 2)$.

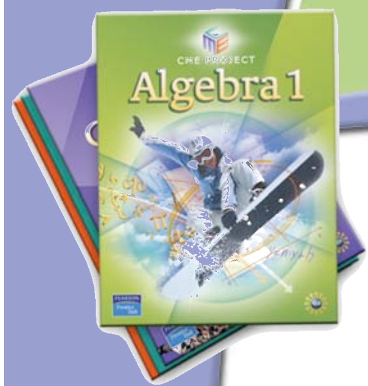
The remainder defines the line.



Linking Geometry to Algebra

You can determine the equation of the line tangent to $y = x^3 - x + 1$ at point $(1,1)$ by dividing by $(x - 1)^2$.

The remainder defines the line.



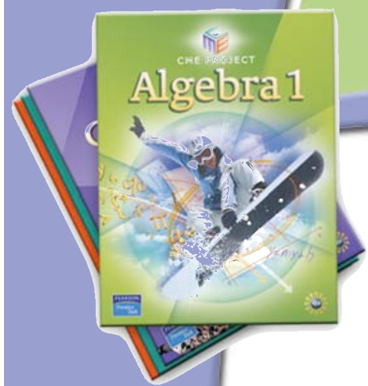
Reducing Computational Overhead

Lagrange Interpolation

- Find a polynomial function that fits this table of data.

1	–5
2	1
5	–29

EDC



Reducing Computational Overhead

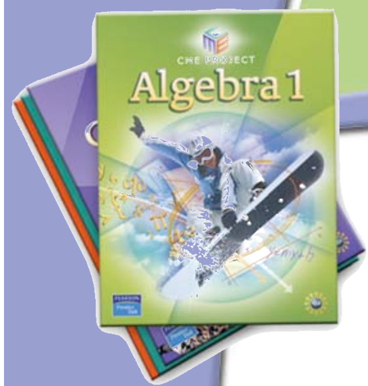
Lagrange Interpolation

- Write the polynomial in this form.

$$f(x) = A(x-2)(x-5) + B(x-1)(x-5) + C(x-1)(x-2)$$

1	–5
2	1
5	–29

EDC



Reducing Computational Overhead

Lagrange Interpolation

$$f(x) = A(x-2)(x-5) + B(x-1)(x-5) + C(x-1)(x-2)$$

- For $f(1)$, the second two terms drop out, so

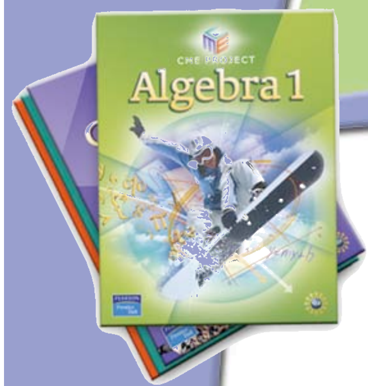
$$f(1) = A(1-2)(1-5)$$

$$-5 = 4A$$

$$A = -\tfrac{5}{4}$$

1	–5
2	1
5	–29

EDC



Reducing Computational Overhead

Lagrange Interpolation

$$f(x) = A(x-2)(x-5) + B(x-1)(x-5) + C(x-1)(x-2)$$

- For $f(2)$, the first and last terms drop out, so

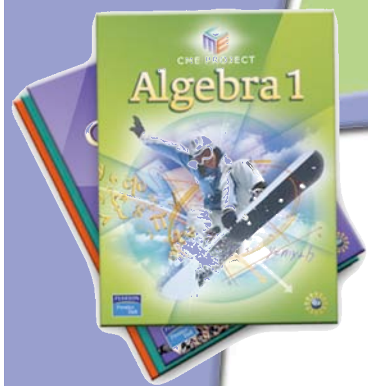
$$f(2) = B(2-1)(2-5)$$

$$1 = -3B$$

$$B = -\tfrac{1}{3}$$

1	–5
2	1
5	–29

EDC



Reducing Computational Overhead

Lagrange Interpolation

$$f(x) = A(x-2)(x-5) + B(x-1)(x-5) + C(x-1)(x-2)$$

- For $f(5)$, the last two terms drop out, so

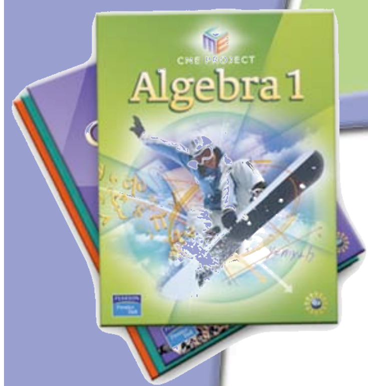
$$f(5) = C(5-1)(5-2)$$

$$-29 = 12C$$

$$C = -\tfrac{29}{12}$$

1	–5
2	1
5	–29

EDC



Reducing Computational Overhead

Lagrange Interpolation

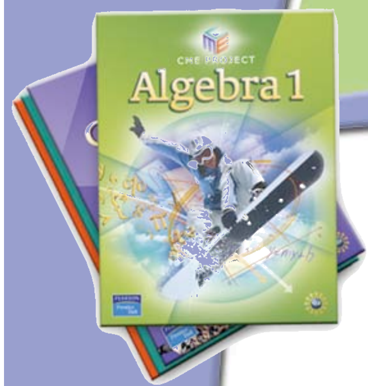
- So the following function fits the table.

$$f(x) = (-\tfrac{5}{4})(x-2)(x-5) + (-\tfrac{1}{3})(x-1)(x-5) + (-\tfrac{29}{12})(x-1)(x-2)$$

- Now, write the function in normal form.

1	–5
2	1
5	–29

EDC



Highlighting Similarities in Algebraic Structures

Find an integer whose remainder is

- 2 when divided by 3,
- 4 when divided by 5, and
- 3 when divided by 7.

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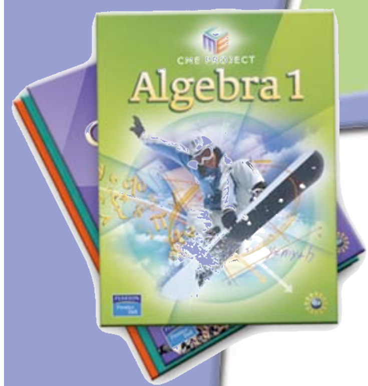


Highlighting Similarities in Algebraic Structures

Write the number as

$$A(5\cdot 7)+B(3\cdot 7)+C(3\cdot 5)$$

Highlighting Similarities in Algebraic Structures

Write the number as

A(35) + B(21) + C(15)

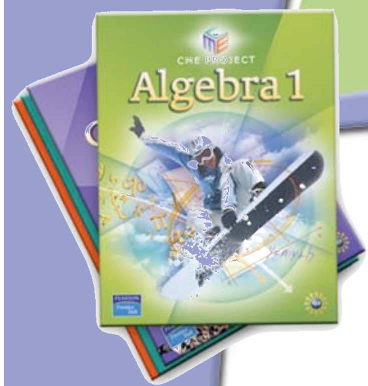
- when divided by 3, the last two terms drop out.
- So 35A / 3 has a remainder of 2.
- A=1 works, since 35 / 3 has remainder 2.

Highlighting Similarities in Algebraic Structures

Write the number as

A(35) + B(21) + C(15)

- when divided by 5, the first and last terms drop out.
- So 21B / 5 has a remainder of 4.
- 21 / 5 has a remainder of 1, so B=4 works, since 84 / 5 has remainder 4.



Highlighting Similarities in Algebraic Structures

Write the number as

A(35) + B(21) + C(15)

- when divided by 7, the first two terms drop out.
- So 15C / 7 has a remainder of 3.
- 15 / 7 has a remainder of 1, so B=3 works, since 45 / 7 has remainder 3.

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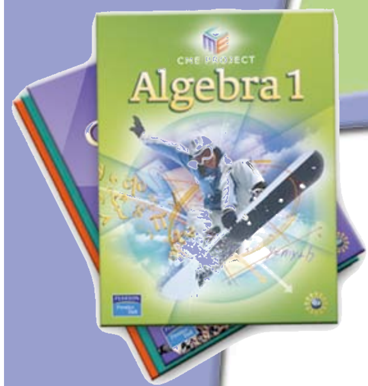
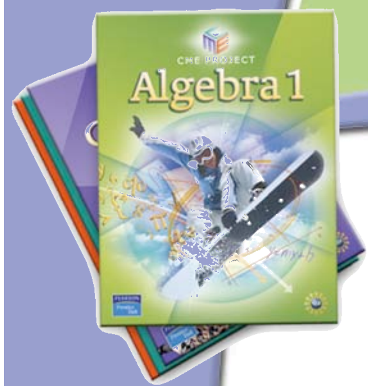


Highlighting Similarities in Algebraic Structures

So the number

$1(35) + 4(21) + 3(15) = 164$

fits the pattern.



Highlighting Similarities in Algebraic Structures

164 is an integer whose remainder is

- 2 when divided by 3,
- 4 when divided by 5, and
- 3 when divided by 7.

But it's not the only one. In fact, for any integer *k*, 164 + 105*k* fits the pattern.

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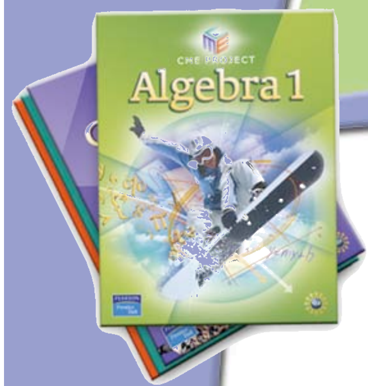
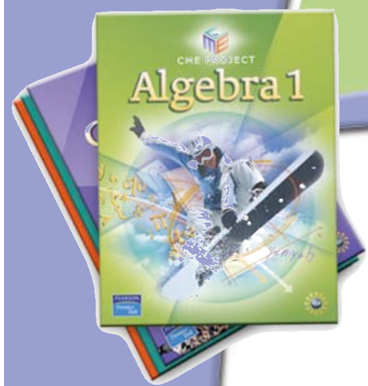


Highlighting Similarities in Algebraic Structures

The process underneath the “Chinese Remainder Theorem” is similar to the concept underneath Lagrange Interpolation



Highlighting Similarities in Algebraic Structures

$f(x) = -4x^2 + 18x - 19$ is a polynomial function such that

- $f(1) = -5$,
- $f(2) = 1$, and
- $f(5) = -29$.

But it's not the only one. In fact, for any real number *a*,

$$f(x) = -4x^2 + 18x - 19 + a(x-1)(x-2)(x-5)$$

fits the pattern.

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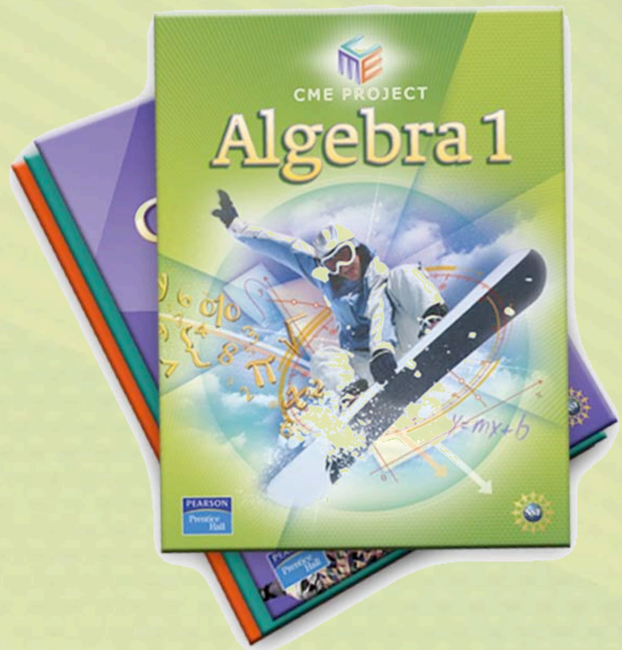
CME Project Availability Dates

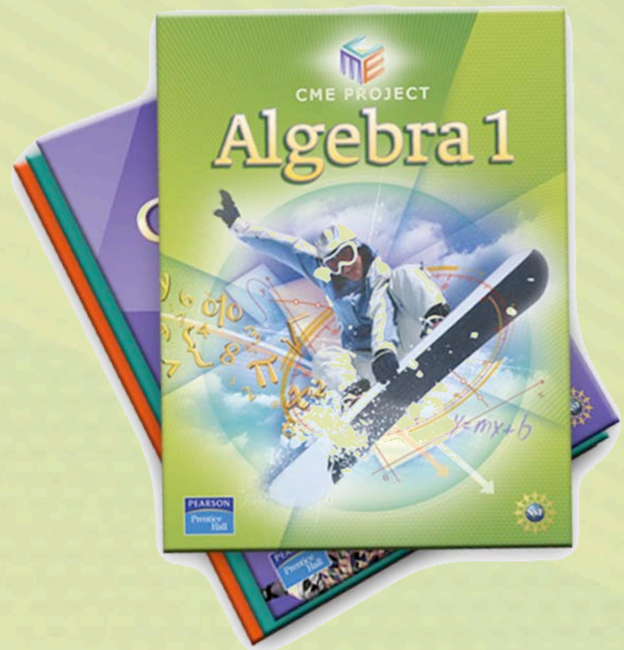
- **Algebra 1 and Geometry**
  - **Available right now!**
- **Algebra 2**
  - **Available Spring 2008**
- **Precalculus**
  - **Available Summer 2008**



CME Project

For more information

- www.edc.org/cmeproject
- www.phschool.org/cme
- Kevin Waterman
  kwaterman@edc.org



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