# The *CME Project*

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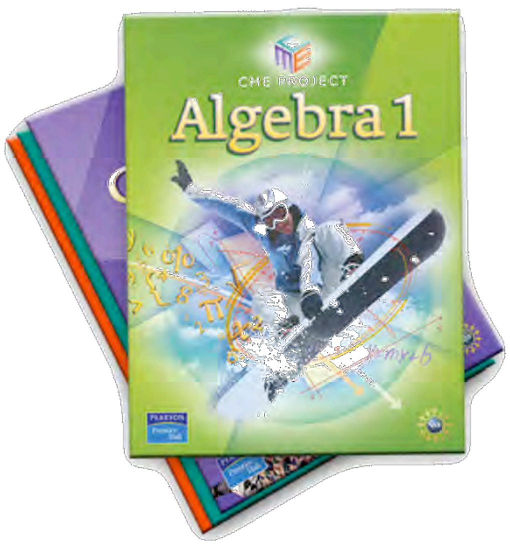
# The *CME Project*

Is a comprehensive four-year NSF-funded high school curriculum developed by Education Development Center, Inc.




under the direction of

## Al Cuoco

forthcoming from Pearson
with additional support from
Texas Instruments

# *The Utility of Mathematics*

Mathematics constitutes one of the most ancient and noble intellectual traditions of humanity. It is an enabling discipline for all of science and technology, providing powerful tools for analytical thought as well as the concepts and language for precise quantitative description of the world around us. It affords knowledge and reasoning of extraordinary subtlety and beauty, even at the most elementary levels.

-RAND Mathematics Study Panel, 2002

# *Fundamental Organizing Principle*

The widespread utility and effectiveness of mathematics come not just from mastering specific skills, topics, and techniques, but more importantly, from developing the ways of thinking

- *the habits of mind* -

used to create the results.

- The *CME Project* Implementation Guide

# *The CME Project Approach*

- "Traditional" course structure -
  familiar but different:
  - Focuses on particular mathematical habits
  - Uses examples and contexts from many fields
  - Organized around mathematical themes of elementary algebra, geometry, advanced algebra, and analysis.
- The program suits teachers who want
  - a problem- and exploration-based curriculum
  - to bring activities to "closure"
  - the familiar US course structure.

# *Core Principles of the Curriculum*

- Is faithful to mathematics as a discipline
- Fosters mathematical habits of mind
- Develops general-purpose mathematical tools
- Experience before formalization
- Separates convention and vocabulary from matters of mathematical substance
- Makes essential use of technology
- Low Threshold-High Ceiling

# *Guess-Check-Generalize*

Useful for solving problems in elementary and more advanced algebra, but also a habit of mind that dovetails with the style of work used by algebraists. It is centered around a fundamental use of abstraction:

finding and exploiting regularity in calculations

Pre-algebra students have no difficulty solving this problem:

“Mary drives from Boston to Washington, a trip of 500 miles. If she travels at an average rate of 60 MPH on the way down and 50 MPH on the way back, how many hours does her trip take?”

But students struggle to solve this problem:

"Mary drives from Boston to Washington, and she travels at an average rate of 60 MPH on the way down and 50 MPH on the way back. If the total trip takes $18\frac{1}{3}$ hours, how far is Boston from Washington?"

The Guess-Check-Generalize method in this case:

- Guess at an answer, for example, 300.
- Take the guess, divide it by 60

  5
- Take the guess, divide it by 50

  6
- Add your answers and see if you get 18 and $\frac{1}{3}$

  5+6= 11 which is not 18 and $\frac{1}{3}$
- Repeat this process for another random guess
- Now generalize this “checking” process

This process leads to the idea:

$$\frac{guess}{60} + \frac{guess}{50} = 18\frac{1}{3}$$

which helps students come up with the equation

$$\frac{x}{60} + \frac{x}{50} = 18\frac{1}{3}$$

This can now be solved with basic algebra

$$5x + 6x = 18 \cdot 300 + 1 \cdot 100$$

$$11x = 5500$$

$$x = 500$$

# Equation of a Line and the *Point-Tester Method*

Assumption: Three points *A*, *B*, and *C* lie on the same line if and only if

$$m(A, B) = m(B, C)$$

Problem: Find an equation of the line through *A=(3, -1)* and *B=(5, 3)*.

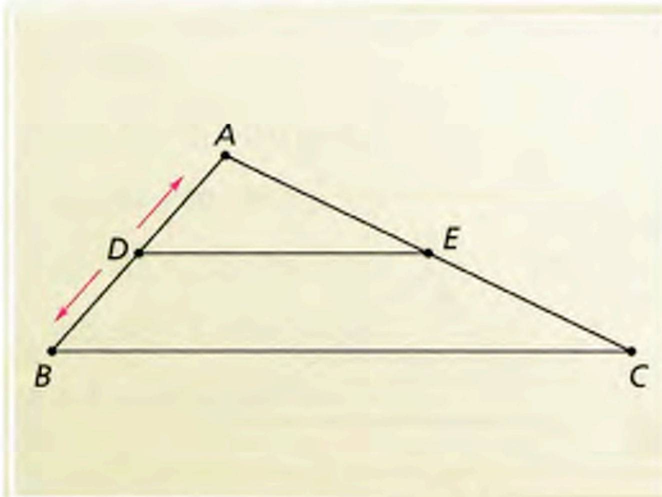
Point-testing can lead to the equation: $2 = \dfrac{y-3}{x-5}$

# Use of Technology

## Geometry Software

### Part 2 Splitting Two Sides of a Triangle

Use geometry software. Draw $\triangle ABC$. Place a point $D$ anywhere on side $\overline{AB}$. Then construct a segment $\overline{DE}$ that is parallel to $\overline{BC}$.



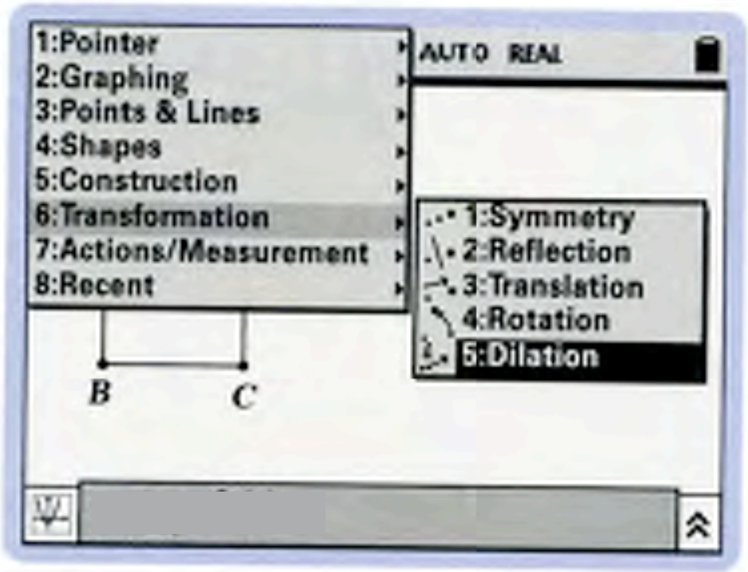

Drag point $D$ along $\overline{AB}$.

$\triangle ADE$ and $\triangle ABC$ are a pair of nested triangles.

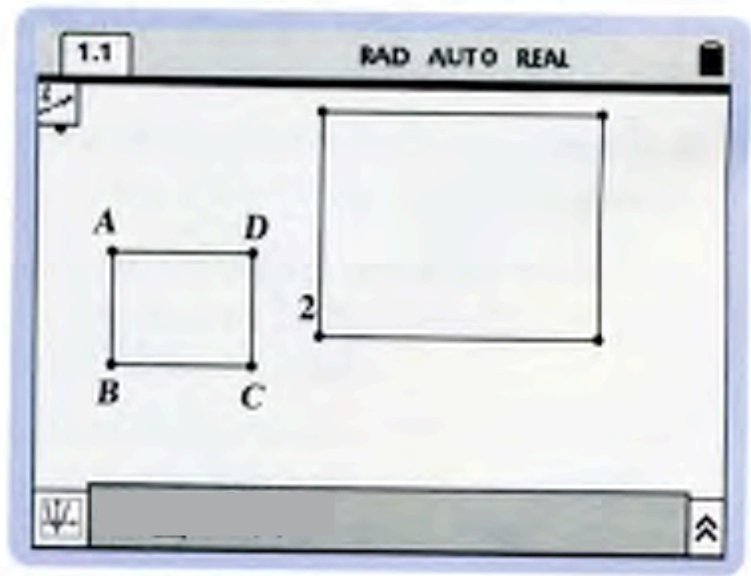
6. Use the software to find the ratio $\frac{AD}{AB}$.
7. Find two other length ratios with the same value. Do all three ratios remain equal to each other when you drag point $D$ along $\overline{AB}$?
8. As you drag $D$ along $\overline{AB}$, describe what happens to the figure. Make a conjecture about the effect of $\overline{DE}$ being parallel to $\overline{BC}$.

# Use of Technology

Choose **Dilation** in the **Transformation** menu.




Select the scale factor on the screen. Then select the rectangle. The scaled rectangle appears on the screen. Press enter to anchor the scaled figure.



# …Back to the Side-Splitter Theorems

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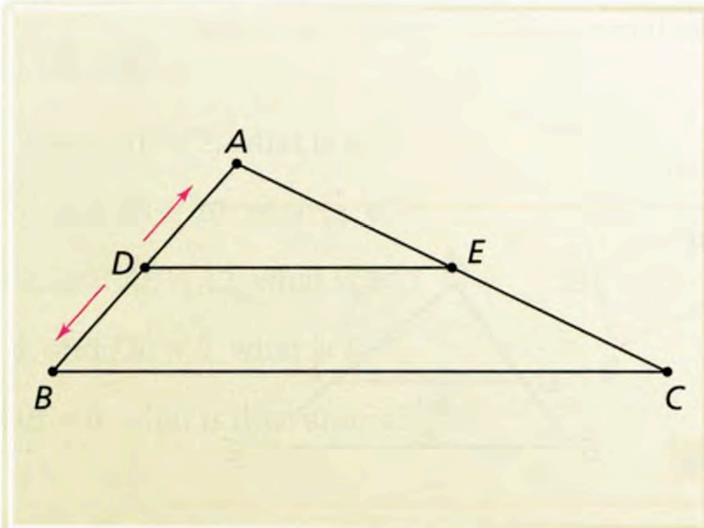


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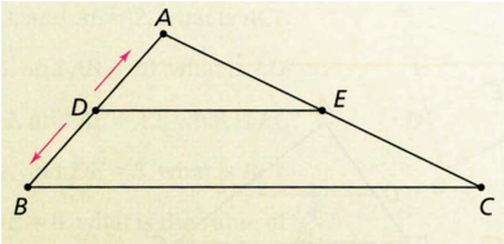



# Dialogues

## Minds in Action

**Tony and Sasha are finishing the In-Class Experiment.**

**Tony** I like dragging points on the computer screen and watching what happens.

**Sasha** Me too. Triangle $ADE$ and the ratios all got small when we dragged $D$ close to $A$.

**Tony** And when we dragged $D$ close to $B$, the two triangles were almost one, and the ratios were almost 1!

**Sasha** The two triangles always had the same shape too. I think that happened because we constructed $\overline{DE}$ parallel to $\overline{BC}$.

**Tony** The parallel segment seemed to make everything work nicely.

**Sasha** So, can we make a conjecture about what having a parallel segment like $\overline{DE}$ does for the figure?

**Tony** Can we say something like "A parallel-to-one-side segment inside a triangle makes two proportional triangles"?

**Sasha** Hmm. I get the idea. I think we have to work on the wording.

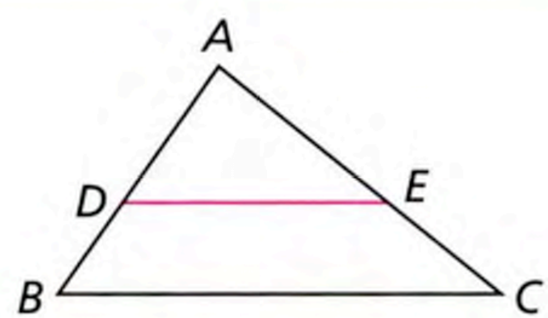
# Definitions as Capstones

## Definitions

In $\triangle ABC$ with $D$ on $\overline{AB}$ and $E$ on $\overline{AC}$, $\overline{DE}$ **splits two sides proportionally** ($\overline{AB}$ and $\overline{AC}$) if and only if

$$\frac{AB}{AD} = \frac{AC}{AE}$$

The ratio $\frac{AB}{AD}$ is called the **common ratio.**

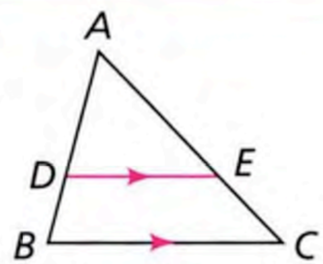


# Theorems as Milestones

## Theorem 4.1 *The Parallel Side-Splitter Theorem*

If a segment with endpoints on two sides of a triangle is parallel to the third side of the triangle, then

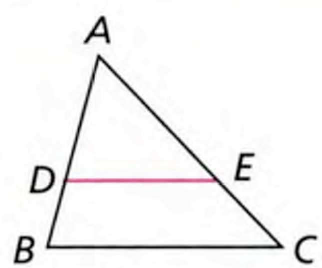
- the segment splits the sides it intersects proportionally
- the ratio of the length of the parallel side to the length of this segment is the common ratio




$$\frac{AB}{AD} = \frac{AC}{AE} = \frac{BC}{DE}$$

## Theorem 4.2 *The Proportional Side-Splitter Theorem*

If a segment with endpoints on two sides of a triangle splits those sides proportionally, then the segment is parallel to the third side.




$$\frac{AB}{AD} = \frac{AC}{AE} \Rightarrow \overline{DE} \parallel \overline{BC}$$

# Proof
## Another General-Purpose Tool

Experience $\rightarrow$ Intuition $\rightarrow$ Formalization

How do you come up with the *idea* for a proof?

The CME Project recognizes that finding the sequence of ideas that constitute a proof is the difficult task. The written proof is just a record.

## 4.11 Proving the Side-Splitter Theorems

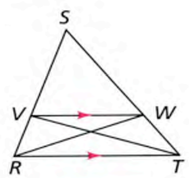
Recall what the first part of the Parallel Side-Splitter Theorem says:

> If a segment with endpoints on two sides of a triangle is parallel to the third side of the triangle, then it splits the sides it intersects proportionally.

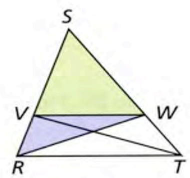
To prove this theorem, you can show that, in the figure below, if $\overline{VW} \parallel \overline{RT}$, then $\frac{SV}{VR} = \frac{SW}{WT}$.

In the figures below, triangles $SVW$ and $RVW$ have the same height. This means that the ratio of their areas is equal to the ratio of their base lengths, $SV$ and $VR$.

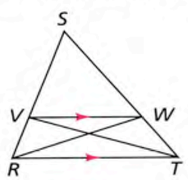
This fact comes from Exercise 2 in Lesson 4.9.



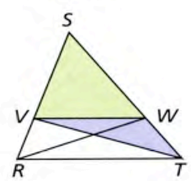

$$\frac{\text{area}(\triangle SVW)}{\text{area}(\triangle RVW)} = \frac{SV}{VR}$$




In the figures below, triangles $SVW$ and $TVW$ have the same height. Thus, the ratio of their areas is equal to the ratio of their base lengths, $SW$ and $WT$.




$$\frac{\text{area}(\triangle SVW)}{\text{area}(\triangle TVW)} = \frac{SW}{WT}$$




You have two fractions with the same numerator,

$$\frac{\text{area}(\triangle SVW)}{\text{area}(\triangle RVW)} \text{ and } \frac{\text{area}(\triangle SVW)}{\text{area}(\triangle TVW)}.$$

The denominators aren't the same, but are they equal? Recall Problem 1 from Lesson 4.9 Getting Started. Triangles $RVW$ and $TVW$ share the same base $\overline{VW}$. They have the same height since $\overline{VW}$ is parallel to $\overline{RT}$. So they have the same area.

You can combine all of these results to draw a conclusion about $SV$, $VR$, $SW$, and $WT$.

$$\frac{SV}{VR} = \frac{\text{area}(\triangle SVW)}{\text{area}(\triangle RVW)} = \frac{\text{area}(\triangle SVW)}{\text{area}(\triangle TVW)} = \frac{SW}{WT}$$

### For You to Do

1. Using what has been outlined in this lesson, write a complete proof of the first part of the Parallel Side-Splitter Theorem.

**Habits of Mind**

**Understand the Process** A "tidy" proof like this does not necessarily happen easily. You build it from many notes, sketches, erasures, more sketches, more notes, and a lot of talking to yourself!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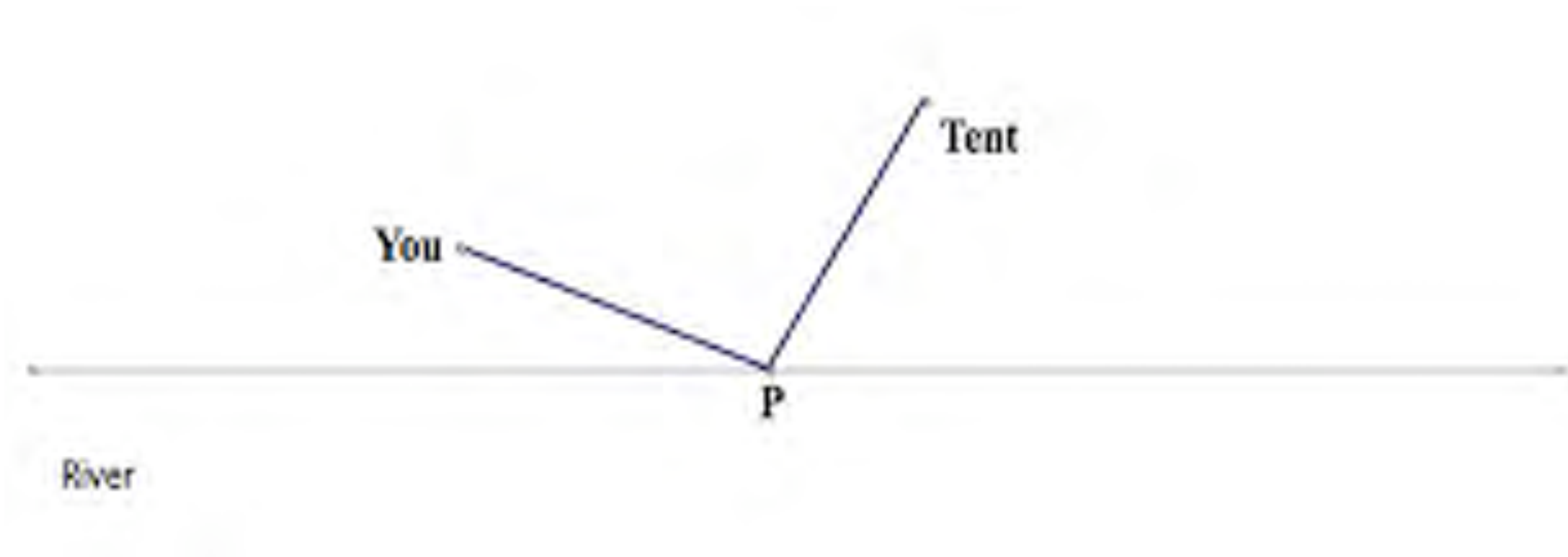
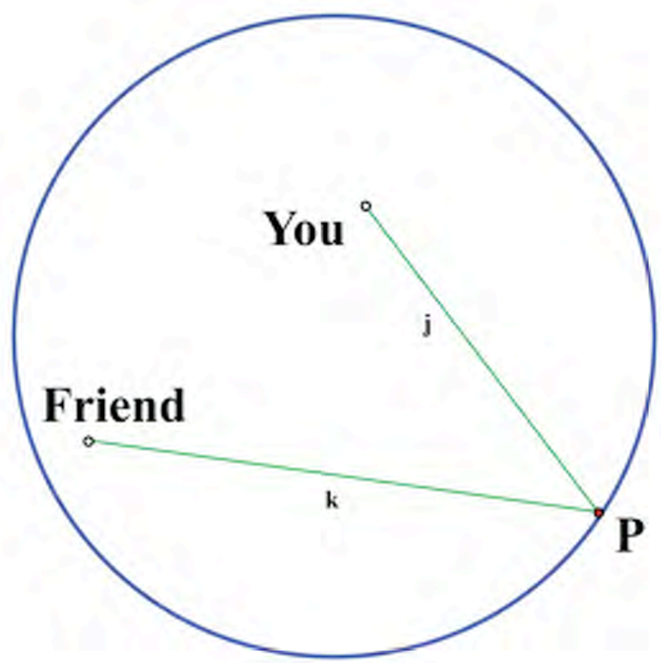
# Low Threshold, High Ceiling

- High expectations for all students
- An example: the *burning tent problem*

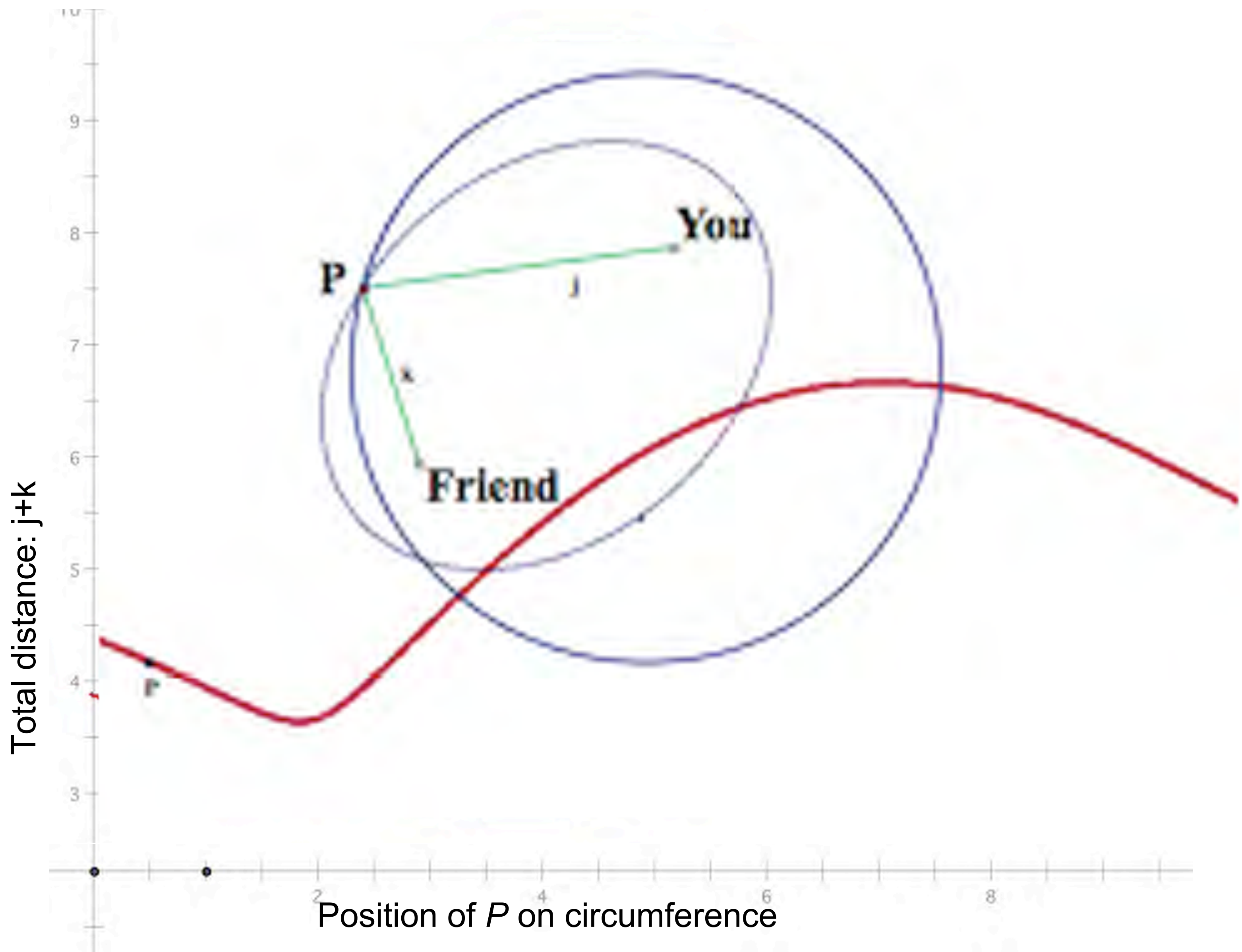


# The Sunglasses Problem

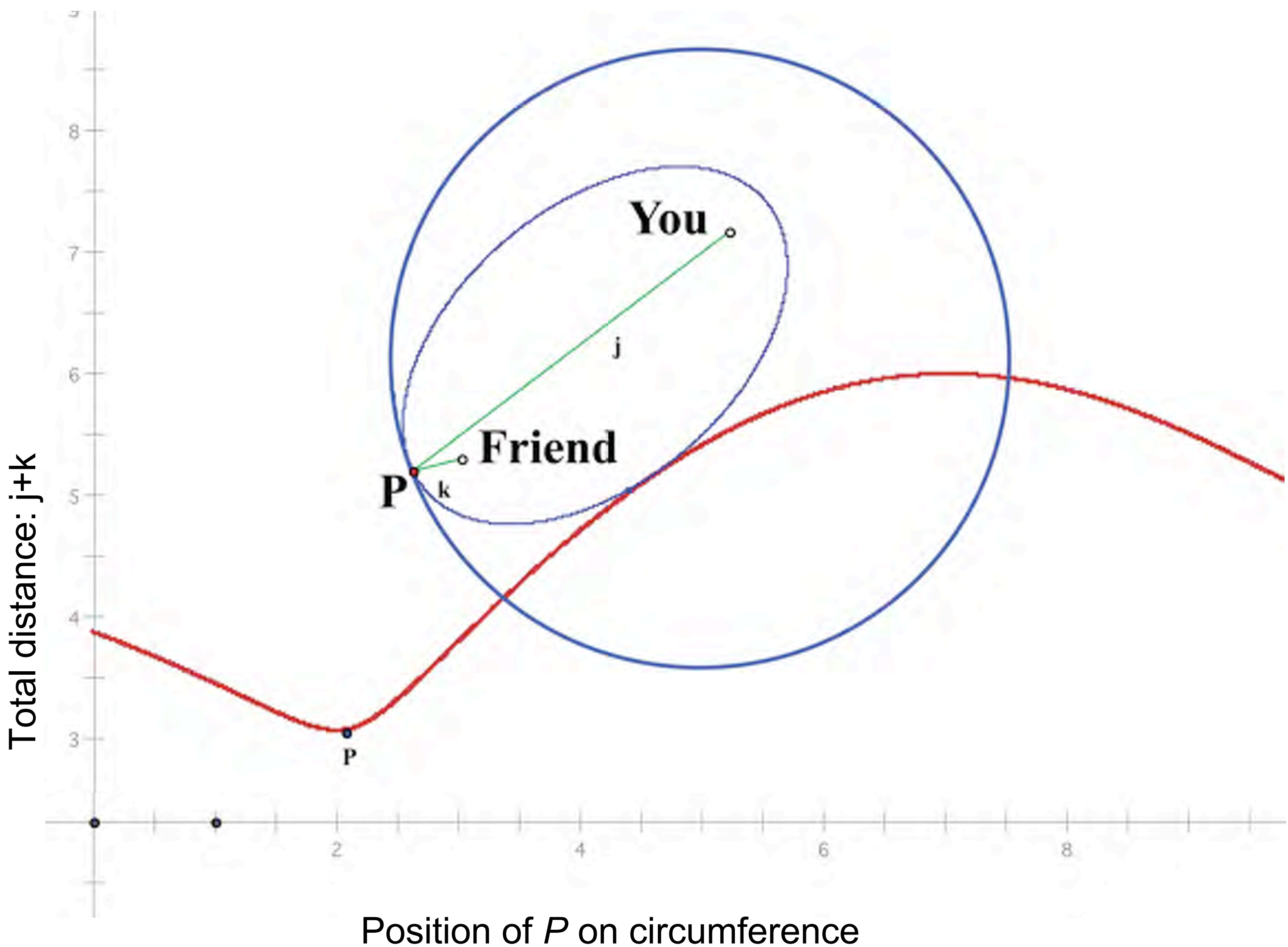
"You are in a circular swimming pool and you want to swim to the edge of the pool to drop off your sunglasses, and then swim to your friend. Where should you land on the edge of the pool to keep the trip to a minimum?"



The total distance (j+k)
at a random point P on the circle



The optimal choice of P
(for this configuration of “You” and “Friend”)



## Examples of Mathematical Habits of Mind

- Guess-Check-Generalize
- Point-Tester Method
- Reasoning by Continuity

## Other Features

- Use of Technology
- Dialogues
- Experience before Formalization
- Low Threshold-High Ceiling

For additional information please visit the

CME Project's websites:

http://www.edc.org/cmeproject

and

http://PHSchool.com/cme

or for questions contact:

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