



CME PROJECT

Ideas from the newest NSF
high school mathematics program

from **EDC** and Pearson

New York City, 2007

Writer's-cramp saver

- I talk fast.
- The PowerPoint will (soon) be available
- Go to <http://www.edc.org/CME> and click on CME Showcase

What *is* geometry?

- **As a mathematical discipline:**
Seeing, describing, measuring, and reasoning logically about shape and space
- **As seen in state tests and in texts *before* HS:**
oodles of words for objects and features about which one has little or nothing to say; arbitrary formulas for measurement.
- **As seen in state tests and in texts *in* HS:**
oodles of theorems about which one has little to say; arbitrary forms for proof.

How do we satisfy tests *and* math?

- *Same content*, arranged to tell a better story
- Math is not just the *results* of mathematical thinking, but the *ways* of thinking that produce these results: “habits of mind”

Mathematical “habits of mind”?

- Looking for what *doesn't* change: invariants
- Reasoning by continuity
- Looking for connections
- ...

Invariants

- Numerical invariants

x	y
3	6
8	16
5.5	11

- Geometric invariants: example (quadrilateral with midpoints)

Mathematical “habits of mind”?

- Looking for what *doesn't* change: invariants
- Reasoning by continuity: example (triangle conjecture)
- Looking for connections


How do we satisfy tests *and* math?

- *Same content*, arranged to tell a better story
- Math is not just the *results* of mathematical thinking, but the *ways* of thinking that produce these results: “habits of mind”
- Proof: less about *form* of a logical argument, more about *content*;
less about how to *present* the argument, more about *how to find it*.

Mathematical “habits of mind”?

- Looking for what *doesn't* change: invariants
- Reasoning by continuity
- Looking for connections:
This is what proof is all about.


Measuring in 2-D—What is area?

- Um, well, uh....
- But whatever it is, if two figures are “the same” they have the same area. 
- And so, if we cut and rearrange the parts, area doesn't change.

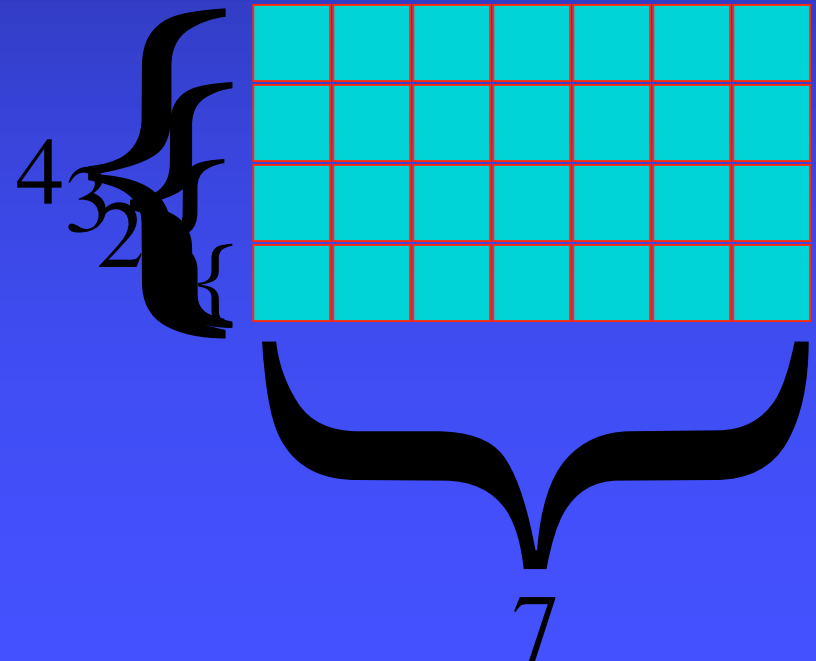


- That's what we *mean* by area: the invariant under dissection. (Hilbert)

Area is amount of (2-D) “stuff”

If  is the unit of “stuff,”
then,

$$\text{Area} = 4 \times 7$$



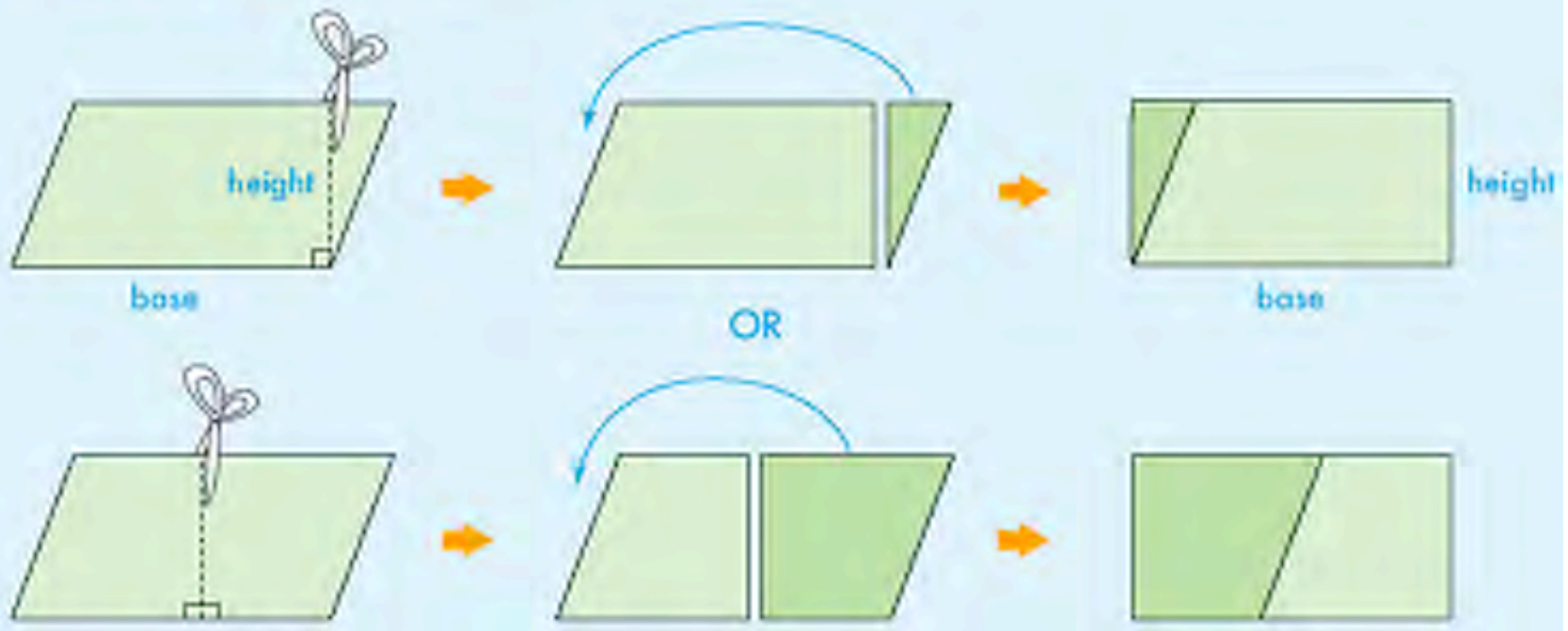
Inventing area formulas

■ *Area of rectangle = base \times height*

■ So...

Area of parallelogram = base \times height

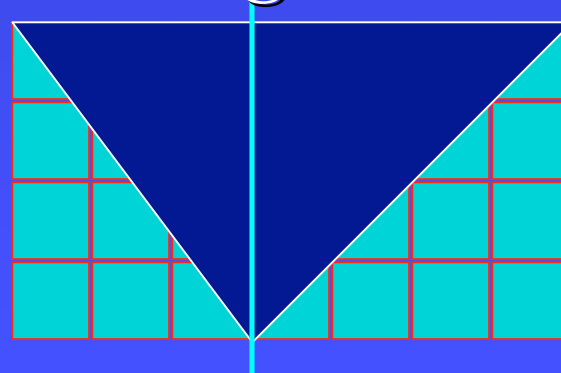
AREA OF A PARALLELOGRAM



Area is amount of (2-D) “stuff”

What is the area of the blue triangle?

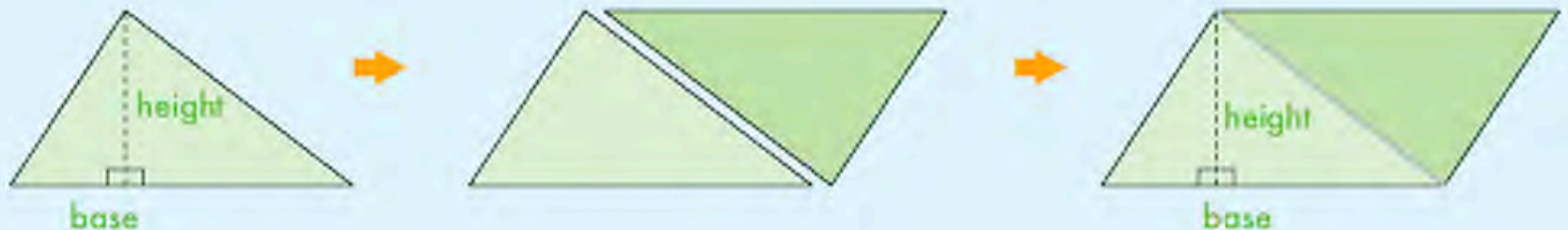
- *Area of whole rectangle* = 4×7
- *Area of left-side rectangle* = 4×3
- *Area of right-side rectangle* = 4×4
- *Area of left-side triangle* = $1/2$ of 4×3
- *Area of right-side triangle* = $1/2$ of 4×4
- *Area of whole triangle* = $1/2$ of 4×7



Inventing area formulas

- Two congruent triangles form a parallelogram
- *Area of parallelogram = base \times height*
- So...
Area of triangle = $1/2$ base \times height

AREA OF A TRIANGLE



Inventing area formulas

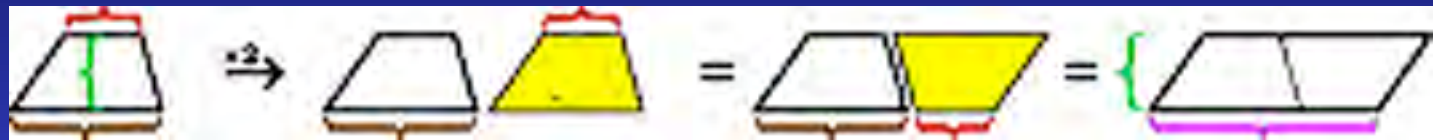
- *Slice parallel to base at half the height*
- *Fit the top triangle to the bottom trapezoid*



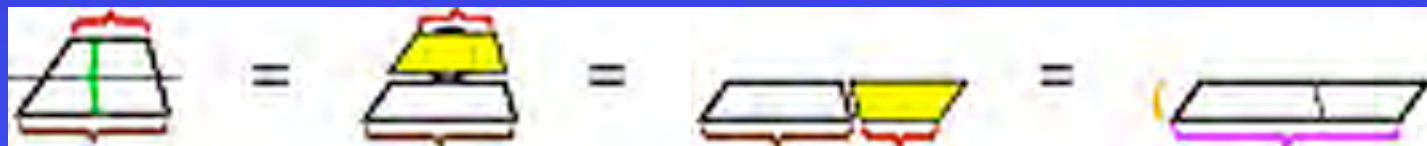
- So...
Area of triangle = base \times 1/2 height

Inventing area formulas

- Inventing a method for dissecting trapezoids



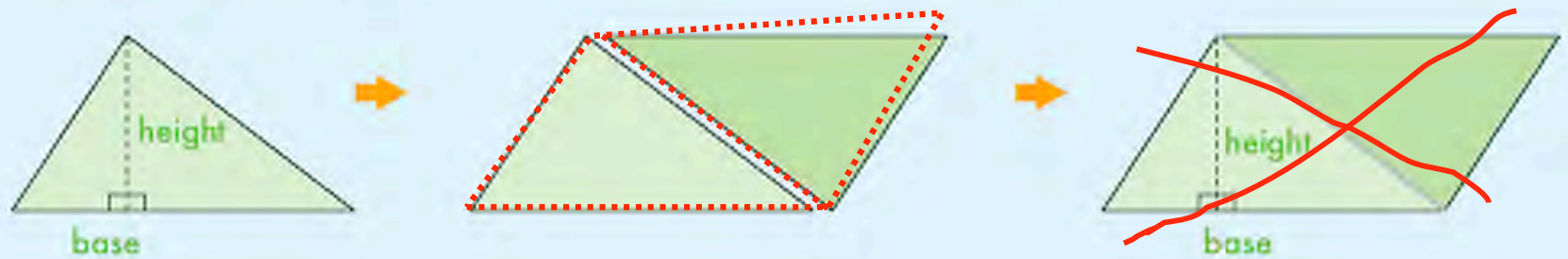
- $A(\text{parallelogram}) = [(base_1 + base_2) \times height]$
- $A(\text{trapezoid}) = 1/2 [(base_1 + base_2) \times height]$



- $A(\text{trapezoid}) = (base_1 + base_2) \times 1/2 height$

But do the pictures work?!

AREA OF A TRIANGLE



- How can we prove that the parts fit right?



Mathematical “habits of mind”?

- Looking for what *doesn't* change: invariants
- Reasoning by continuity
- Looking for connections:
The “burning tent” problem.

Connecting geometry to...

- Functions, conic sections, optimization:
The burning tent problem

Why a new math series?

- Innovative teaching; familiar structure Inverness
- Disciplinary focus
- Mathematicians & teachers
- Classroom-effective methods
- Mathematics at the center;
students bringing math into their world Applications
- High expectations for students;
total respect for teachers Teaching is hard enough!
- No special-purpose clutter; solid methods

Thank you!

- E. Paul Goldenberg
- <http://www2.edc.org/CME>