High School Teaching: Standards, Practices, and Habits of Mind

Al Cuoco

Joint Mathematics Meetings, 2013

Outline

Outline

The Habits of Mind Approach

What mathematicians most wanted and needed from me was to learn my ways of thinking, and not in fact to learn my proof of the geometrization conjecture for Haken manifolds.

—William Thurston
"On Proof and Progress in Mathematics."
Bulletin of the American Mathematical Society, 1994

Our Fundamental Organizing Principle

The widespread utility and effectiveness of mathematics come not just from mastering specific skills, topics, and techniques, but more importantly, from developing the ways of thinking—the habits of mind—used to create the results.

—Cuoco, Goldenberg, & Mark
"Habits of Mind: An Organizing Principle for High School Curricula." *The Journal of Mathematical Behavior*, 1996.

THE NOTION OF MATHEMATICAL PRACTICE

It will be helpful to name and (at least partially) specify some of the things—practices, dispositions, sensibilities, habits of mind—entailed in doing mathematics. . . . These are things that mathematicians typically do when they do mathematics. At the same time most of these things, suitably interpreted or adapted, could apply usefully to elementary mathematics no less than to research.

—Hyman Bass

"A Vignette of Doing Mathematics." *The Montana Mathematics Enthusiast*, 2011.

The Notion of Mathematical Practice

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise. . . . Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

—CCSS, 2010

OUTLINE

Common Core: Mathematical Practices

Eight attributes of mathematical proficiency:

1. **Make sense of complex problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Two Emergent Problems

Two Emergent Problems

1. These standards have been treated in isolation from each other and from the content standards.

Two Emergent Problems

1. These standards have been treated in isolation from each other and from the content standards.
2. Many practitioners find the statements of 3,7,and 8 obscure.

Two Emergent Problems

1. These standards have been treated in isolation from each other and from the content standards.

2. Many practitioners find the statements of 3,7,and 8 obscure.

Commissioned by the MA Department of Elementary and Secondary Education, we developed a 45 hour course, *Developing Mathematical Practice* aimed at helping high school teachers implement these standards across the entire high school program.

THE DMP COURSE

The course is designed to show how the standards for mathematical practice

- enhance the teaching and learning of standard content by making it
  - more understandable to students
  - easier to teach
  - more satisfying to all involved

THE DMP COURSE

The course is designed to show how the standards for mathematical practice

- enhance the teaching and learning of standard content by making it
  - more understandable to students
  - easier to teach
  - more satisfying to all involved
- form a web of practices, all connected in inseparable ways

The DMP Course

The course is designed to show how the standards for mathematical practice

- enhance the teaching and learning of standard content by making it
  - more understandable to students
  - easier to teach
  - more satisfying to all involved
- form a web of practices, all connected in inseparable ways
- bring coherence and parsimony to the zoo of topics and methods in a standard high school program

The DMP Course

The course is designed to show how the standards for mathematical practice

- enhance the teaching and learning of standard content by making it
  - more understandable to students
  - easier to teach
  - more satisfying to all involved
- form a web of practices, all connected in inseparable ways
- bring coherence and parsimony to the zoo of topics and methods in a standard high school program

The course is now a regular offering of ESE and EDC, conducted around the country and taught by teams of teachers.

Outline

Sample Agenda

Day 1

8:00: Coffee, welcome, gossip

8:27: Getting Started

- A rectangle has perimeter 32. What could its area be?

9:32: Introductions, and discuss the standards for mathematical practice

9:59: Break

10:12: SSS and Area

11:32: Lunch

12:17: Geometric Optimization-1

- Of all rectangles of a given perimeter, which maximizes area?

1:42: Geometric Optimization-2

- The "Burning Tent" problem

2:47: Break

3:02: Debrief

3:30: End

Days 2 and 3

Day 2: Abstracting generality from repeated reasoning

- Word problems
- Graphing
- Heron's formula
- Modeling functions
- Lines of best fit

Day 3: Structure

- Integers and Polynomials
- Fitting functions to tables
- Factoring
- Quadratics
- Monthly payments
- Probability distributions
- Complex numbers

Days 4 and 5

Day 4: Viable arguments

- Area and dissections
- Congruence → area → similarity
- Critiquing proofs
- Reasoning about irrational numbers
- Extending definitions
- Critiquing the reasoning of others
- Mathematical induction

Day 5: Implications for High School

- Designing activities for students
- The PARCC Content Frameworks
- End with some lovely mathematics

Example: Maximizing Area

1. A rectangle has perimeter 32. What could its area be?

Example: Maximizing Area

1. A rectangle has perimeter 32. What could its area be?
2. Show how to cut a 4×12 rectangle into a square with the same perimeter.

Example: Maximizing Area

1. A rectangle has perimeter 32. What could its area be?
2. Show how to cut a 4×12 rectangle into a square with the same perimeter.
3. Show how to cut a 5×11 rectangle into a square with the same perimeter.

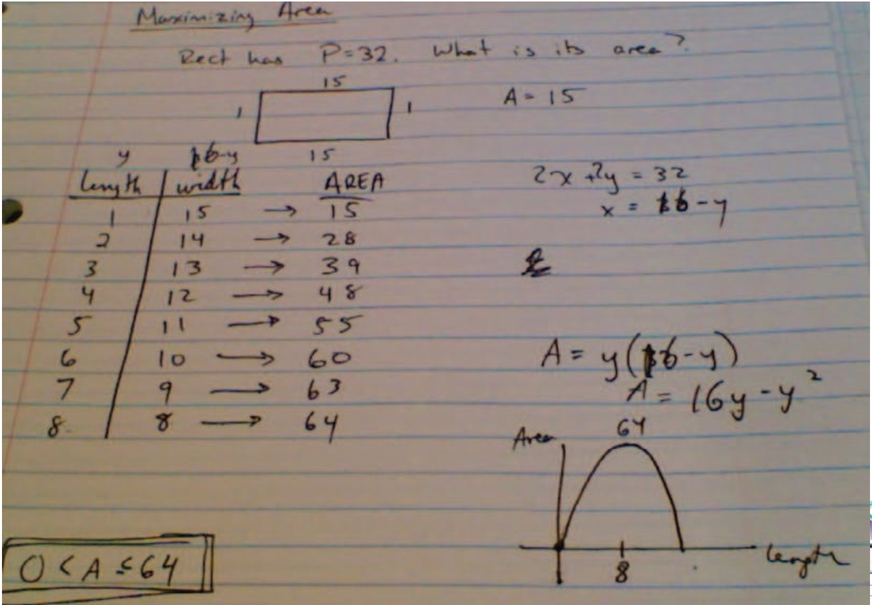
EXAMPLE: MAXIMIZING AREA

1. A rectangle has perimeter 32. What could its area be?
2. Show how to cut a 4×12 rectangle into a square with the same perimeter.
3. Show how to cut a 5×11 rectangle into a square with the same perimeter.
4. Show how to cut a 6×10 rectangle into a square with the same perimeter.

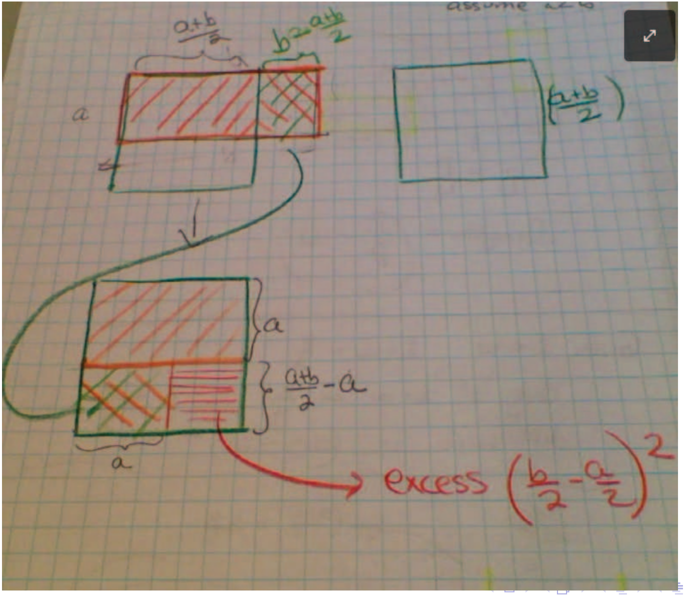
Example: Maximizing Area

1. A rectangle has perimeter 32. What could its area be?
2. Show how to cut a 4×12 rectangle into a square with the same perimeter.
3. Show how to cut a 5×11 rectangle into a square with the same perimeter.
4. Show how to cut a 6×10 rectangle into a square with the same perimeter.
5. Show how to cut an $a \times b$ rectangle into a square with the same perimeter.

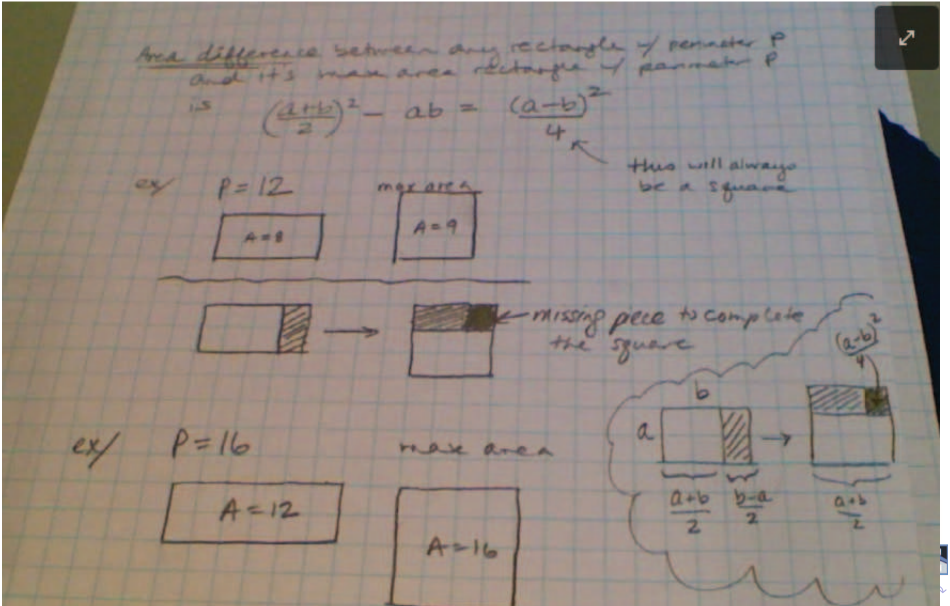
Example: Maximizing Area



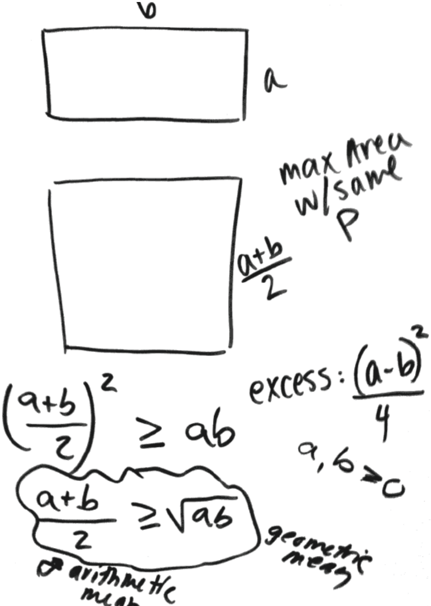
Example: Maximizing Area



Example: Maximizing Area



Example: Maximizing Area



EXAMPLE: BUILDING EQUATIONS
WORD PROBLEMS

Mary drives from Boston to Chicago, and she travels at an average rate of 60 MPH on the way down and 50 MPH on the way back. *If the total trip takes 36 hours, how far is Boston from Chicago?*

Example: Building Equations

Word Problems

Mary drives from Boston to Chicago, and she travels at an average rate of 60 MPH on the way down and 50 MPH on the way back. *If the total trip takes 36 hours, how far is Boston from Chicago?*

Mary drives from Boston to Chicago, and she travels at an average rate of 60 MPH on the way down and 50 MPH on the way back. *If Boston is 1000 miles from Chicago, how long did the trip take?*

EXAMPLE: BUILDING EQUATIONS
WORD PROBLEMS

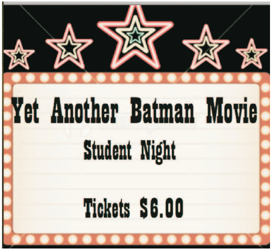
Mary drives from Boston to Chicago, and she travels at an average rate of 60 MPH on the way down and 50 MPH on the way back. *If the total trip takes 36 hours, how far is Boston from Chicago?*

Mary drives from Boston to Chicago, and she travels at an average rate of 60 MPH on the way down and 50 MPH on the way back. *If Boston is 1000 miles from Chicago, how long did the trip take?*

"The difficulty lies in setting up the equation, not solving it."

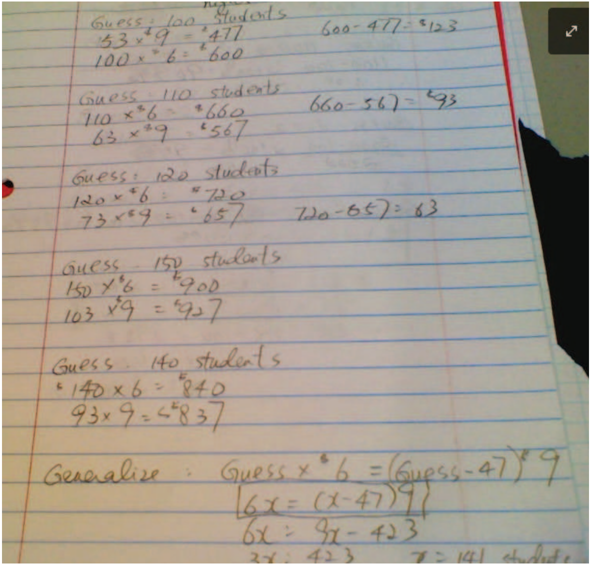
Example: Building Equations

Mr. Brennan is looking at a theater marquis:



He says “Isn’t that great. The theater usually charges $9.00 per person. On Student Night, I can bring 47 more students for the same cost.” How many students can Mr. Brennan bring to the theater on Student Night?

EXAMPLE: BUILDING EQUATIONS



Example: Equations for Lines

EXAMPLE: EQUATIONS FOR LINES

The phenomenon was first noticed in precalculus . . .

Example: Equations for Lines

The phenomenon was first noticed in precalculus . . .
Graph

$$16\,x^2 - 96\,x + 25\,y^2 - 100\,y - 156 = 0$$

Example: Equations for Lines

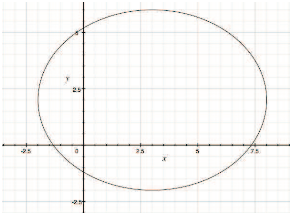
The phenomenon was first noticed in precalculus . . .

Graph

$$16\,x^2 - 96\,x + 25\,y^2 - 100\,y - 156 = 0$$

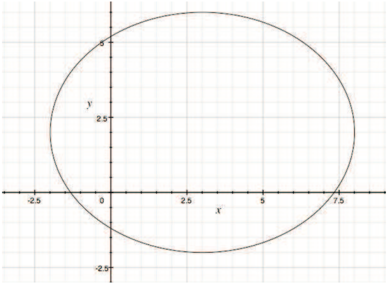
$$16\,x^2 - 96\,x + 25\,y^2 - 100\,y - 156 = 0 \Rightarrow \frac{(x-3)^2}{25} + \frac{(y-2)^2}{16} = 1$$

$$\frac{(x-3)^2}{25} + \frac{(y-2)^2}{16} = 1 \Rightarrow$$



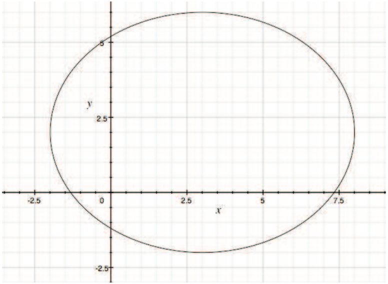
EXAMPLE: EQUATIONS FOR LINES

$$\frac{(x-3)^2}{25} + \frac{(y-2)^2}{16} = 1$$



Example: Equations for Lines

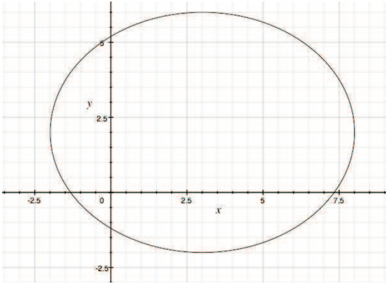
$$\frac{(x-3)^2}{25} + \frac{(y-2)^2}{16} = 1$$



Is $(7.5, 3.75)$ on the graph?

Example: Equations for Lines

$$\frac{(x-3)^2}{25} + \frac{(y-2)^2}{16} = 1$$



Is $(7.5, 3.75)$ on the graph?

This led to the idea that "equations are point testers."

Example: Equations for Lines

- Suppose a student, new to algebra and with no formulas in tow, is asked to find the equation of the vertical line ℓ that passes through $(5, 4)$.

Example: Equations for Lines

- Suppose a student, new to algebra and with no formulas in tow, is asked to find the equation of the vertical line ℓ that passes through $(5, 4)$.
- Students can draw the line, and, just as in the word problem example, they can guess at some points and check to see if they are on ℓ.

Example: Equations for Lines

- Suppose a student, new to algebra and with no formulas in tow, is asked to find the equation of the vertical line ℓ that passes through $(5, 4)$.
- Students can draw the line, and, just as in the word problem example, they can guess at some points and check to see if they are on ℓ.
- Trying some points like $(5, 1)$, $(3, 4)$, $(2, 2)$, and $(5, 17)$ leads to a generic guess-checker:

EXAMPLE: EQUATIONS FOR LINES

- Suppose a student, new to algebra and with no formulas in tow, is asked to find the equation of the vertical line ℓ that passes through $(5, 4)$.
- Students can draw the line, and, just as in the word problem example, they can guess at some points and check to see if they are on ℓ.
- Trying some points like $(5, 1)$, $(3, 4)$, $(2, 2)$, and $(5, 17)$ leads to a generic guess-checker:

  To see if a point is on ℓ, you check that its x-coordinate is 5.

Example: Equations for Lines

- Suppose a student, new to algebra and with no formulas in tow, is asked to find the equation of the vertical line ℓ that passes through $(5, 4)$.
- Students can draw the line, and, just as in the word problem example, they can guess at some points and check to see if they are on ℓ.
- Trying some points like $(5, 1)$, $(3, 4)$, $(2, 2)$, and $(5, 17)$ leads to a generic guess-checker:

  To see if a point is on ℓ, you check that its x-coordinate is 5.

- This leads to a guess-checker: $x \stackrel{?}{=} 5$ and the equation

$$x = 5$$

EXAMPLE: EQUATIONS FOR LINES

- What about lines for which there is no simple guess-checker? The idea is to find a geometric characterization of such a line and then to develop a guess-checker based on that characterization. One such characterization uses *slope*.

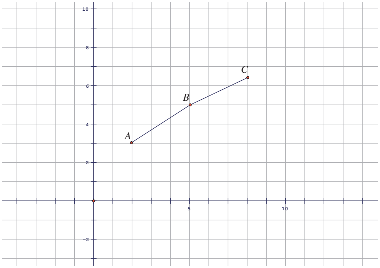
EXAMPLE: EQUATIONS FOR LINES

- What about lines for which there is no simple guess-checker? The idea is to find a geometric characterization of such a line and then to develop a guess-checker based on that characterization. One such characterization uses *slope*.
- In first-year algebra, students study slope, and one fact about slope that often comes up is that three points on the coordinate plane, not all on the same vertical line, are collinear if and only if the slope between any two of them is the same.

EXAMPLE: EQUATIONS FOR LINES

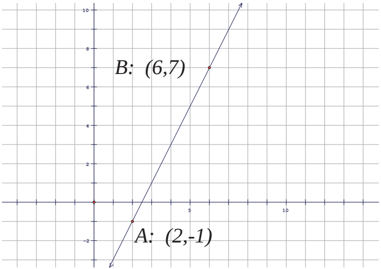
If we let $m(A, B)$ denote the slope between A and B (calculated as change in y-height divided by change in x-run), then the collinearity condition can be stated like this:

Basic assumption: A, B, and C are collinear $\Leftrightarrow m(A, B) = m(B, C)$



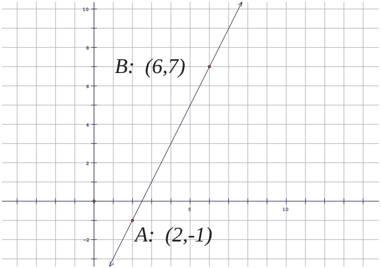
Example: Equations for Lines

What is an equation for $\ell = \overleftrightarrow{AB}$ if $A = (2, -1)$ and $B = (6, 7)$?



EXAMPLE: EQUATIONS FOR LINES

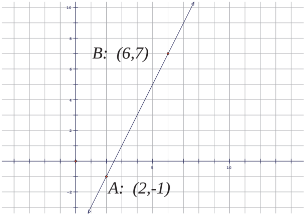
What is an equation for $\ell = \overleftrightarrow{AB}$ if $A = (2, -1)$ and $B = (6, 7)$?



Try some points, keeping track of the steps...

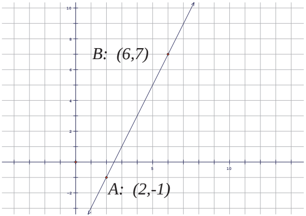
Example: Equations for Lines

- $A = (2, -1)$ and $B = (6, 7)$
- $m(A, B) = 2$



Example: Equations for Lines

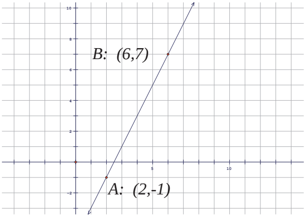
- $A = (2, -1)$ and $B = (6, 7)$
- $m(A, B) = 2$



- Test $C = (3, 4)$:
  $m(C, B) = \frac{4-7}{3-6} \stackrel{?}{=} 2 \Rightarrow$ Nope

Example: Equations for Lines

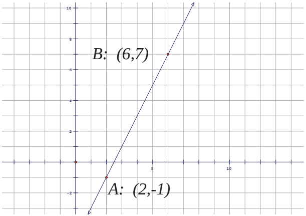
- $A = (2, -1)$ and $B = (6, 7)$
- $m(A, B) = 2$



- Test $C = (3, 4)$: $m(C, B) = \frac{4-7}{3-6} \stackrel{?}{=} 2 \Rightarrow$ Nope
- Test $D = (5, 5)$: $m(D, B) = \frac{5-7}{5-6} \stackrel{?}{=} 2 \Rightarrow$ Yup

Example: Equations for Lines

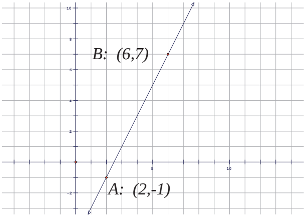
- $A = (2, -1)$ and $B = (6, 7)$
- $m(A, B) = 2$



- Test $C = (3, 4)$:
  $m(C, B) = \frac{4-7}{3-6} \stackrel{?}{=} 2 \Rightarrow$ Nope
- Test $D = (5, 5)$:
  $m(D, B) = \frac{5-7}{5-6} \stackrel{?}{=} 2 \Rightarrow$ Yup
- The "guess-checker?"
  Test $P = (x, y)$:
  $m(P, B) = \frac{y-7}{x-6} \stackrel{?}{=} 2$

Example: Equations for Lines

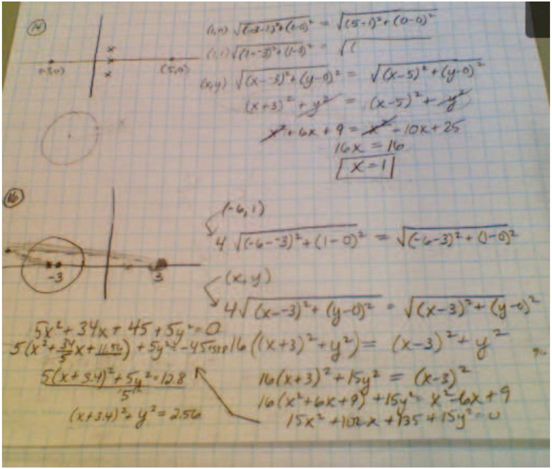
- $A = (2, -1)$ and $B = (6, 7)$
- $m(A, B) = 2$



- Test $C = (3, 4)$: $m(C, B) = \frac{4-7}{3-6} \stackrel{?}{=} 2 \Rightarrow$ Nope
- Test $D = (5, 5)$: $m(D, B) = \frac{5-7}{5-6} \stackrel{?}{=} 2 \Rightarrow$ Yup
- The "guess-checker?" Test $P = (x, y)$: $m(P, B) = \frac{y-7}{x-6} \stackrel{?}{=} 2$

And an equation is $\frac{y-7}{x-6} = 2$

Example: Other Loci



Other Examples Where This Habit Is Useful

- Finding lines of best fit
- Building expressions ("*three less than a number*")
- Fitting functions to tables of data
- Deriving the quadratic formula
- Establishing identities in Pascal's triangle
- Using recursive definitions in a CAS or spreadsheet

⋮

Example: Factoring

From a Popular Text (∼ 1980)

EXAMPLE: FACTORING

FROM A POPULAR TEXT (∼ 1980)

"Factoring Pattern for $x^2 + bx + c$, c Negative"

EXAMPLE: FACTORING

FROM A POPULAR TEXT (~ 1980)

"Factoring Pattern for $x^2 + bx + c$, c Negative"

Factor. Check by multiplying factors. If the polynomial is not factorable, write "prime."

Example: Factoring

From a Popular Text (∼ 1980)

“Factoring Pattern for $x^2 + bx + c$, c Negative”

Factor. Check by multiplying factors. If the polynomial is not factorable, write “prime.”

1. $a^2 + 4a - 5$	**2.** $x^2 - 2x - 3$	**3.** $y^2 - 5y - 6$
4. $b^2 + 2b - 15$	**5.** $c^2 - 11c - 10$	**6.** $r^2 - 16r - 28$
7. $x^2 - 6x - 18$	**8.** $y^2 - 10c - 24$	**9.** $a^2 + 2a - 35$
10. $k^2 - 2k - 20$	**11.** $z^2 + 5z - 36$	**12.** $r^2 - 3r - 40$
13. $p^2 - 4p - 21$	**14.** $a^2 + 3a - 54$	**15.** $y^2 - 5y - 30$
16. $z^2 - z - 72$	**17.** $a^2 - ab - 30b^2$	**18.** $k^2 - 11kd - 60d^2$
19. $p^2 - 5pq - 50q^2$	**20.** $a^2 - 4ab - 77b^2$	**21.** $y^2 - 2yz - 3z^2$
22. $s^2 + 14st - 72t^2$	**23.** $x^2 - 9xy - 22y^2$	**24.** $p^2 - pq - 72q^2$

Example: Factoring

From a Published Text (2010)

Example: Factoring

From a Published Text (2010)

To factor a trinomial of the form $ax^2 + bx + c$ where $a > 0$, follow these steps:

Example: Factoring

From a Published Text (2010)

To factor a trinomial of the form $ax^2 + bx + c$ where $a > 0$, follow these steps:

A	B	C
F	H	D
G	I	E

Example: Factoring

From a Published Text (2010)

To factor a trinomial of the form $ax^2 + bx + c$ where $a > 0$, follow these steps:

A	B	C
F	H	D
G	I	E

1. Identify the values of a, b, c. Put a in Box A and c in Box B. Put the product of a and c in Box C.
2. List the factors of the number from Box C and identify the pair whose sum is b. Put the two factors you find in Box D and E.
3. Find the greatest common factor of Boxes A [sic] and E and put it in box G.

Example: Factoring
From a Published Text (2010)

A	B	C
F	H	D
G	I	E

4. In Box F, place the number you multiply by Box G to get Box A.
5. In Box H, place the number you multiply by Box F to get Box D.
6. In Box I, place the number you multiply by Box G to get Box E.

Solution: The binomial factors whose product gives the trinomial are $(Fx + I)(Gx + H)$.

Example: Factoring Using the Structure of Expressions

Example: Factoring Using the Structure of Expressions

Factoring monic quadratics:

"Sum-Product" problems

$$x^2 + 14x + 48$$

Example: Factoring Using the Structure of Expressions

Factoring monic quadratics:

"Sum-Product" problems

$$x^2 + 14x + 48$$

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

so...

Example: Factoring Using the Structure of Expressions

Factoring monic quadratics:

"Sum-Product" problems

$$x^2 + 14x + 48$$

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

so...

Find two numbers whose sum is 14 and whose product is 48.

Example: Factoring Using the Structure of Expressions

Factoring monic quadratics:

"Sum-Product" problems

$$x^2 + 14x + 48$$

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

so...

Find two numbers whose sum is 14 and whose product is 48.

$$(x + 6)(x + 8)$$

Example: Factoring Using the Structure of Expressions

What about this one?

$$49x^2 + 35x + 6$$

Example: Factoring Using the Structure of Expressions

What about this one?

$$49x^2 + 35x + 6$$

$$49x^2 + 35x + 6 = (7x)^2 + 5(7x) + 6$$

Example: Factoring Using the Structure of Expressions

What about this one?

$$49x^2 + 35x + 6$$

$$\begin{aligned} 49x^2 + 35x + 6 &= (7x)^2 + 5(7x) + 6 \\ &= \clubsuit^2 + 5\clubsuit + 6 \end{aligned}$$

Example: Factoring Using the Structure of Expressions

What about this one?

$$49x^2 + 35x + 6$$

$$\begin{aligned} 49x^2 + 35x + 6 &= (7x)^2 + 5(7x) + 6 \\ &= \clubsuit^2 + 5\clubsuit + 6 \\ &= (\clubsuit + 3)(\clubsuit + 2) \end{aligned}$$

Example: Factoring Using the Structure of Expressions

What about this one?

$$49x^2 + 35x + 6$$

$$\begin{aligned} 49x^2 + 35x + 6 &= (7x)^2 + 5(7x) + 6 \\ &= \clubsuit^2 + 5\clubsuit + 6 \\ &= (\clubsuit + 3)(\clubsuit + 2) \\ &= (7x + 3)(7x + 2) \end{aligned}$$

Example: Factoring Using the Structure of Expressions

What about this one?

$$6x^2 + 31x + 35$$

Example: Factoring Using the Structure of Expressions

What about this one?

$$6x^2 + 31x + 35$$

$$6(6x^2 + 31x + 35) = (6x)^2 + 31(6x) + 210$$

Example: Factoring Using the Structure of Expressions

What about this one?

$$6x^2 + 31x + 35$$

$$\begin{aligned} 6(6x^2 + 31x + 35) &= (6x)^2 + 31(6x) + 210 \\ &= \clubsuit^2 + 31\clubsuit + 210 \end{aligned}$$

Example: Factoring Using the Structure of Expressions

What about this one?

$$6x^2 + 31x + 35$$

$$\begin{aligned} 6(6x^2 + 31x + 35) &= (6x)^2 + 31(6x) + 210 \\ &= \clubsuit^2 + 31\clubsuit + 210 \\ &= (\clubsuit + 21)(\clubsuit + 10) \end{aligned}$$

Example: Factoring Using the Structure of Expressions

What about this one?

$$6x^2 + 31x + 35$$

$$\begin{aligned} 6(6x^2 + 31x + 35) &= (6x)^2 + 31(6x) + 210 \\ &= \clubsuit^2 + 31\clubsuit + 210 \\ &= (\clubsuit + 21)(\clubsuit + 10) \\ &= (6x + 21)(6x + 10) \end{aligned}$$

Example: Factoring Using the Structure of Expressions

What about this one?

$$6x^2 + 31x + 35$$

$$\begin{aligned}
6(6x^2 + 31x + 35) &= (6x)^2 + 31(6x) + 210 \\
&= \clubsuit^2 + 31\clubsuit + 210 \\
&= (\clubsuit + 21)(\clubsuit + 10) \\
&= (6x + 21)(6x + 10) \\
&= 3(2x + 7) \cdot 2(3x + 5)
\end{aligned}$$

Example: Factoring Using the Structure of Expressions

What about this one?

$$6x^2 + 31x + 35$$

$$\begin{aligned} 6(6x^2 + 31x + 35) &= (6x)^2 + 31(6x) + 210 \\ &= \clubsuit^2 + 31\clubsuit + 210 \\ &= (\clubsuit + 21)(\clubsuit + 10) \\ &= (6x + 21)(6x + 10) \\ &= 3(2x + 7) \cdot 2(3x + 5) \\ &= 6(2x + 7)(3x + 5) \end{aligned}$$

Example: Factoring Using the Structure of Expressions

What about this one?

$$6x^2 + 31x + 35$$

$$\begin{aligned} 6(6x^2 + 31x + 35) &= (6x)^2 + 31(6x) + 210 \\ &= \clubsuit^2 + 31\clubsuit + 210 \\ &= (\clubsuit + 21)(\clubsuit + 10) \\ &= (6x + 21)(6x + 10) \\ &= 3(2x + 7) \cdot 2(3x + 5) \\ &= 6(2x + 7)(3x + 5) \quad \textit{so} \ldots \end{aligned}$$

$$6(6x^2 + 31x + 35) = 6(2x + 7)(3x + 5)$$

Example: Factoring Using the Structure of Expressions

What about this one?

$$6x^2 + 31x + 35$$

$$\begin{aligned} 6(6x^2 + 31x + 35) &= (6x)^2 + 31(6x) + 210 \\ &= \clubsuit^2 + 31\clubsuit + 210 \\ &= (\clubsuit + 21)(\clubsuit + 10) \\ &= (6x + 21)(6x + 10) \\ &= 3(2x + 7) \cdot 2(3x + 5) \\ &= 6(2x + 7)(3x + 5) \quad \textit{so} \ldots \end{aligned}$$

$$\not{6}(6x^2 + 31x + 35) = \not{6}(2x + 7)(3x + 5)$$

Example: Factoring Using the Structure of Expressions

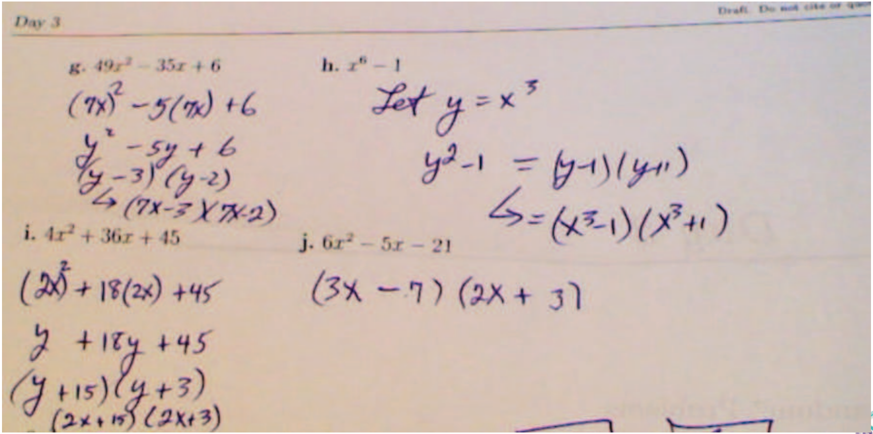
What about this one?

$$6x^2 + 31x + 35$$

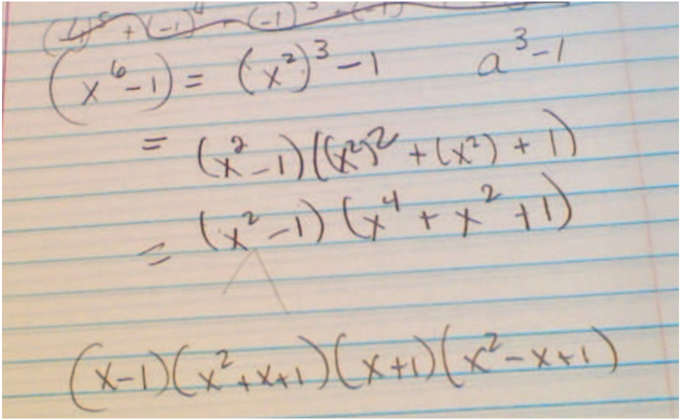
$$\begin{aligned} 6(6x^2 + 31x + 35) &= (6x)^2 + 31(6x) + 210 \\ &= \clubsuit^2 + 31\clubsuit + 210 \\ &= (\clubsuit + 21)(\clubsuit + 10) \\ &= (6x + 21)(6x + 10) \\ &= 3(2x + 7) \cdot 2(3x + 5) \\ &= 6(2x + 7)(3x + 5) \quad \textit{so} \ldots \end{aligned}$$

$$6x^2 + 31x + 35 \;=\; (2x + 7)(3x + 5)$$

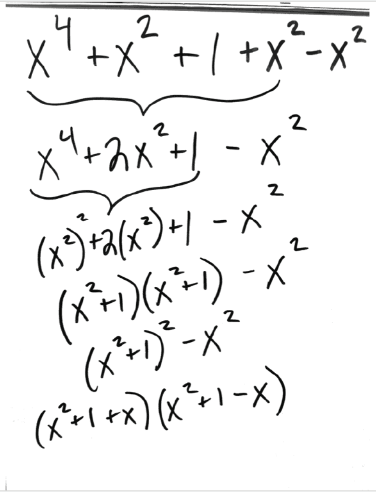
Example: Factoring Using the Structure of Expressions

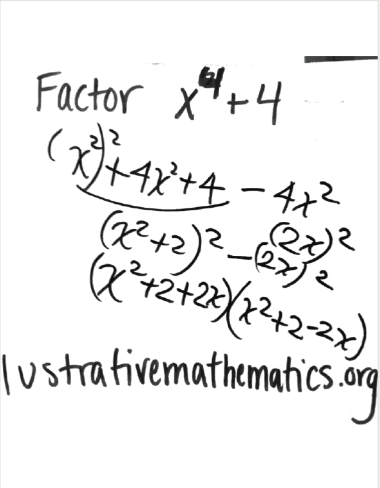


Example: Factoring Using the Structure of Expressions



Example: Factoring Using the Structure of Expressions





Other Examples Where Chunking Is Useful

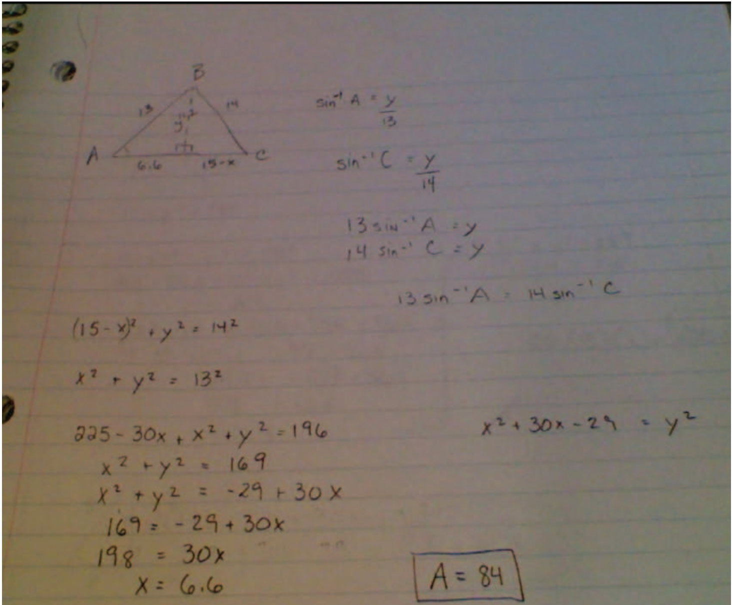
- Completing the square and removing terms
- Solving trig equations
- Analyzing conics and other curves
- All over calculus
- Interpreting results from a computer algebra system

⋮

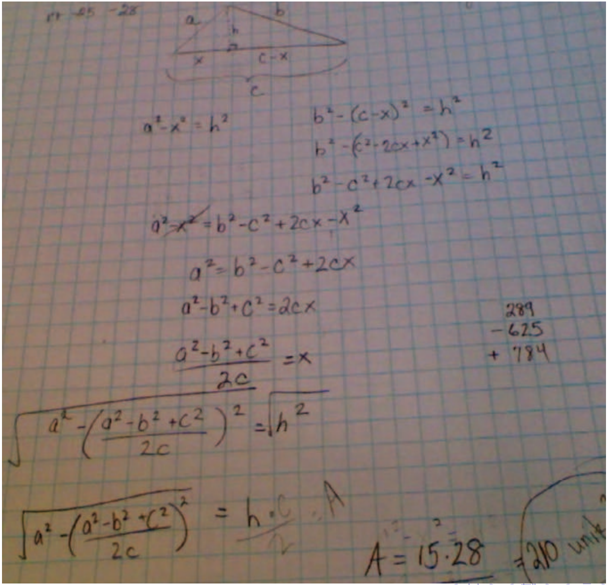
Example: SSS and Area

1. Find the area of a triangle whose side-lengths are 13, 14, 15.
2. Find the area of a triangle whose side-lengths are 6, 25, 29.
3. Find the area of a triangle whose side-lengths are 8, 29, 35.
4. Find the area of a triangle whose side-lengths are 6, 25, 26.
5. Find the area of a triangle whose side-lengths are a, b, c.

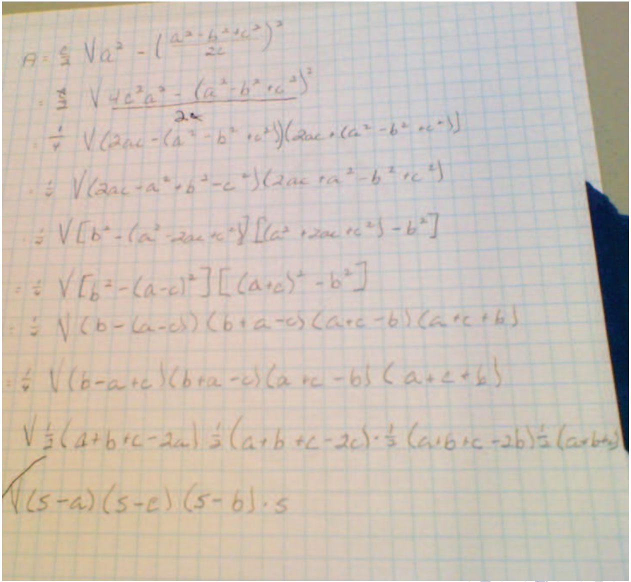
Example: SSS and Area



EXAMPLE: SSS AND AREA



EXAMPLE: SSS AND AREA



Outline

CONCLUSIONS

- There is a practice of mathematics, just as there is a practice of medicine or teaching.

CONCLUSIONS

- There is a practice of mathematics, just as there is a practice of medicine or teaching.
- These Standards for Mathematical Practice capture some essential features of this practice.

CONCLUSIONS

- There is a practice of mathematics, just as there is a practice of medicine or teaching.
- These Standards for Mathematical Practice capture some essential features of this practice.
- Elevating the habits of mind used to create results to the same level of importance as the results themselves can go a long way to connect school mathematics to the real thing.

Conclusions

- There is a practice of mathematics, just as there is a practice of medicine or teaching.
- These Standards for Mathematical Practice capture some essential features of this practice.
- Elevating the habits of mind used to create results to the same level of importance as the results themselves can go a long way to connect school mathematics to the real thing.
- But the Standards for Mathematical Practice will be trivialized if they are not integrated into the Standards for Mathematical Content.

Thanks

acuoco@edc.org

http://mpi.edc.org/dmpsyllabus